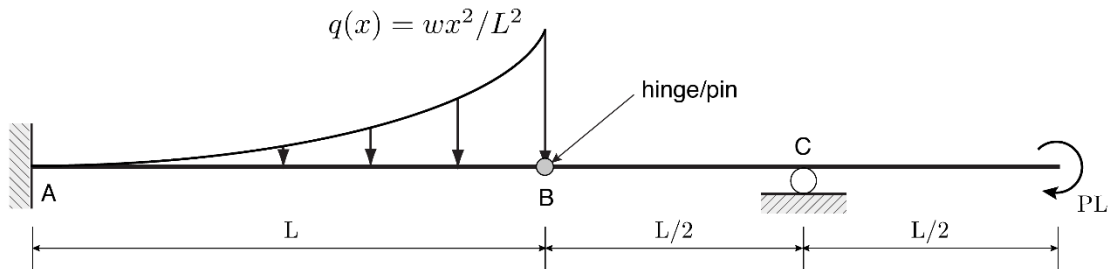


1. (5 pts) Calculate all of the external reactions in this determinate compound beam. Show your results on a free-body diagram (FBD) using arrows and symbolic values.

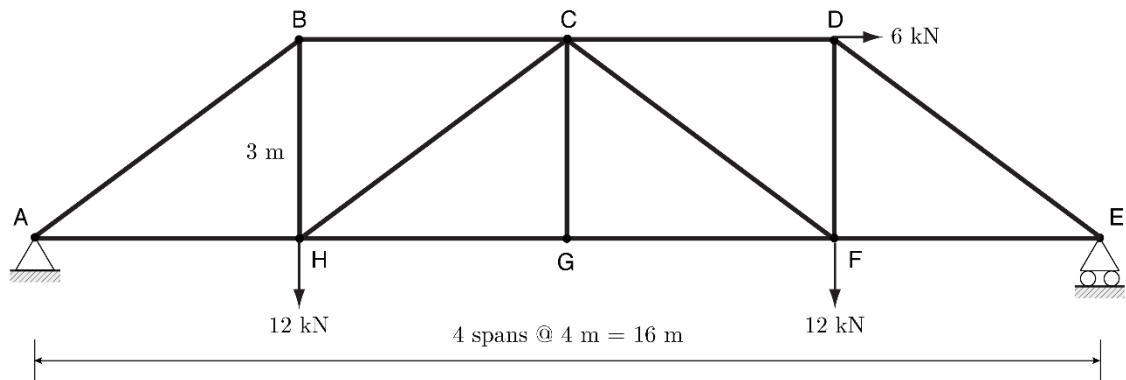


2. Given the planar truss below:

- a. (2 pts) Evaluate if structure is statically determinate.
- b. (2 pts) Identify zero force members (if any).

Calculate bar forces CD, CF, and FG using the following methods and give your results as unsigned numbers and label either tension (T) or compression (C).

- c. (3 pts) Method of Joints.
- d. (3 pts) Method of Sections.



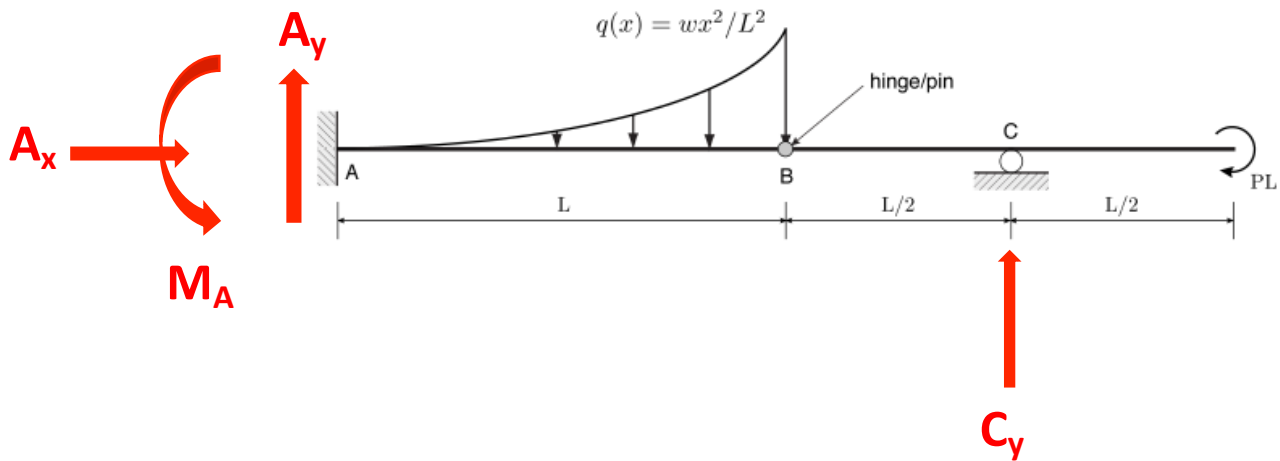
Problem 1

Executive Summary

Problem Statement

Calculate external reactions. Show on FBD.

Results



$$\begin{aligned} A_x &= 0 \\ A_y &= \frac{wL}{3} - 2P \\ M_A &= \frac{wL^2}{4} - 2PL \\ C_y &= 2P \end{aligned}$$

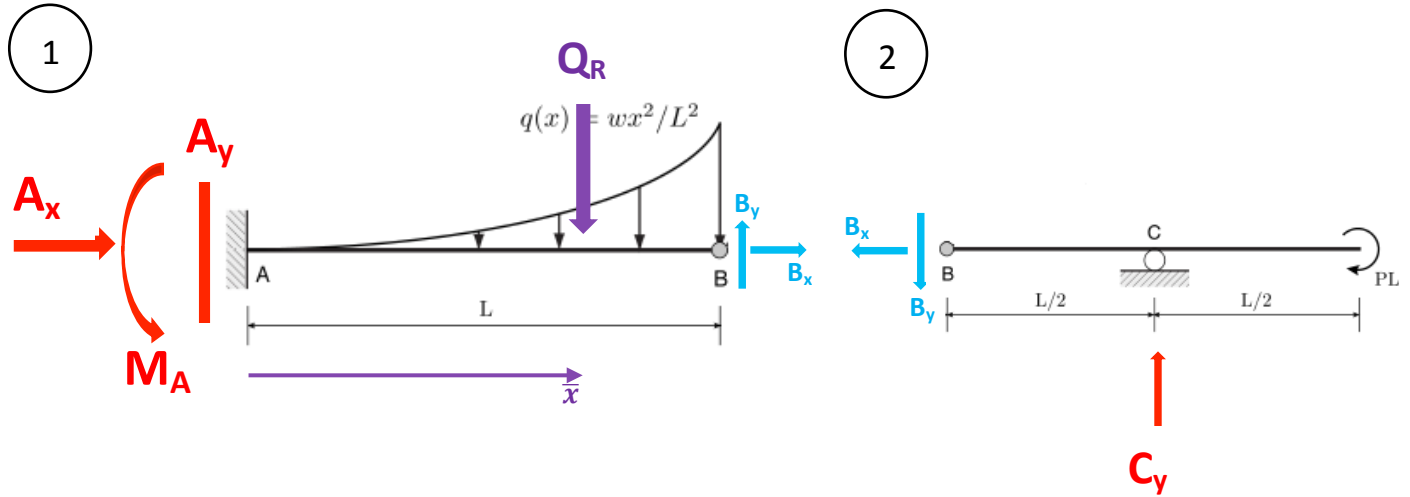
Conclusion

Beam analysis is determinate due to the presence of the hinge/pin at the mid-span. Without the hinge, the beam would be indeterminate to the 1st degree.

Technical Summary

Step 1

Separate beam at hinge into two separate static systems.



Step 2

Solve for resultant force and resultant location of the distributed load.

$$Q_R = \int dA$$
$$Q_R = \int_0^L q(x) dx$$
$$Q_R = \int_0^L \frac{wx^2}{L^2} dx$$
$$Q_R = \frac{wL}{3}$$

$$\bar{x} = \frac{\int x dA}{\int dA}$$
$$\bar{x} = \frac{\left(\int_0^L q(x) x dx \right)}{\left(\int_0^L q(x) dx \right)}$$
$$\bar{x} = \frac{\left(\int_0^L \frac{wx^3}{L^2} dx \right)}{\left(\int_0^L \frac{wx^2}{L^2} dx \right)}$$
$$\bar{x} = \frac{\frac{wL^2}{4}}{\frac{wL}{3}}$$
$$\bar{x} = \frac{3L}{4}$$

Step 3

Solve for hinge force and reactions using equations of equilibrium *for each separate system (alternatively, you can solve for one system, then recombine the compound beam, then solve remaining reactions)*

1

$$\begin{aligned} 4^{\text{th}} \quad \Sigma F_y = 0 &\rightarrow A_y + B_y - Q_R = 0 \\ &\Rightarrow A_y = \frac{wL}{3} - 2P \end{aligned}$$

2

$$\begin{aligned} 1^{\text{st}} \quad \Sigma M_B = 0 &\rightarrow C_y \left(\frac{L}{2}\right) - PL = 0 \\ &\Rightarrow C_y = 2P \end{aligned}$$

5th

$$\begin{aligned} \Sigma M_A = 0 &\rightarrow M_A + B_y(L) - Q_R(\bar{x}) = 0 \\ &\Rightarrow M_A = \frac{wL^2}{4} - 2PL \end{aligned}$$

2nd

$$\begin{aligned} \Sigma F_y = 0 &\rightarrow -B_y + C_y = 0 \\ &\Rightarrow B_y = 2P \end{aligned}$$

6th

$$\begin{aligned} \Sigma F_x = 0 &\rightarrow A_x + B_x = 0 \\ &\Rightarrow A_x = 0 \end{aligned}$$

3rd

$$\begin{aligned} \Sigma F_x = 0 &\rightarrow -B_x = 0 \\ &\Rightarrow B_x = 0 \end{aligned}$$

$$A_x = 0$$

$$A_y = \frac{wL}{3} - 2P$$

$$M_A = \frac{wL^2}{4} - 2PL$$

$$C_y = 2P$$

Problem 2

Executive Summary

Problem Statement

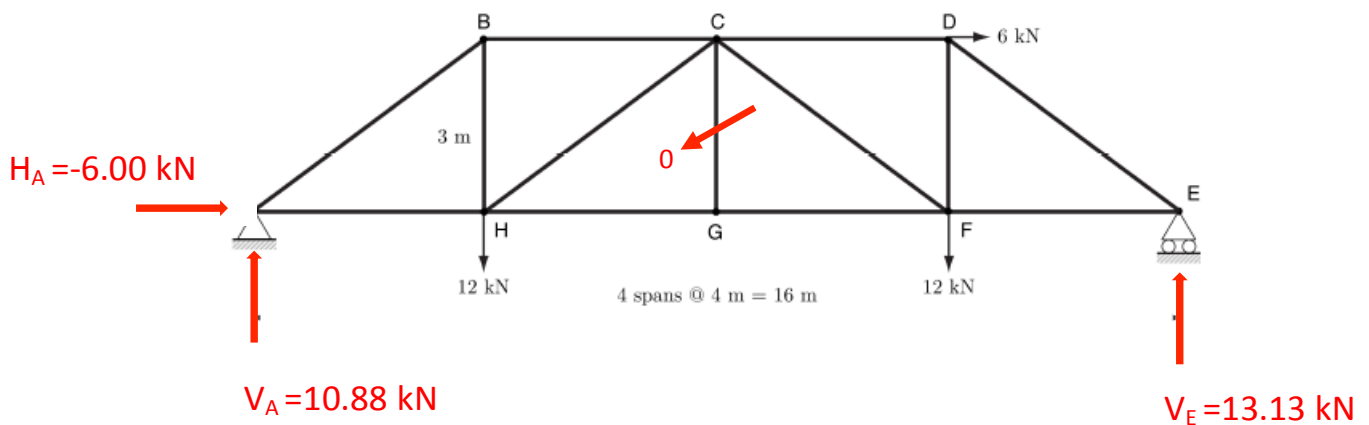
- Evaluate if the structure is statically determinate.
- Identify zero force members.

Calculate bar forces CD, CF, and FG

- Method of Joints
- Method of Sections

Results

Truss is statically determinate



$$\overline{FG} = 19.00 \text{ kN (T)}$$

$$\overline{FC} = 1.88 \text{ kN (C)}$$

$$\overline{DC} = 11.50 \text{ kN (C)}$$

Conclusion

Truss is statically determinate and contains one zero force member. The zero force member provides stability/redundancy and may carry axial force if additional loads are applied.

Technical Summary

Part A

Statically determinate?

Members = $m = 13$

of reactions = $r = 3$

of joints = $j = 8$

$$m + r \leq 2j$$

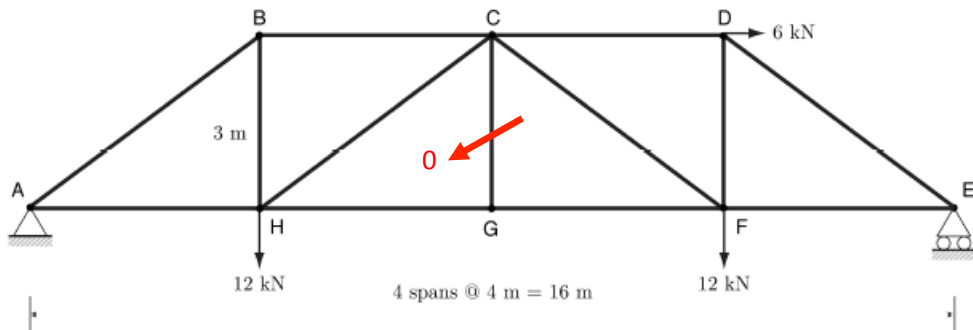
$$13 + 3 \leq 2 * 8$$

$$16 \leq 16 \checkmark$$

Truss is statically determinate

Part B

zero-force members



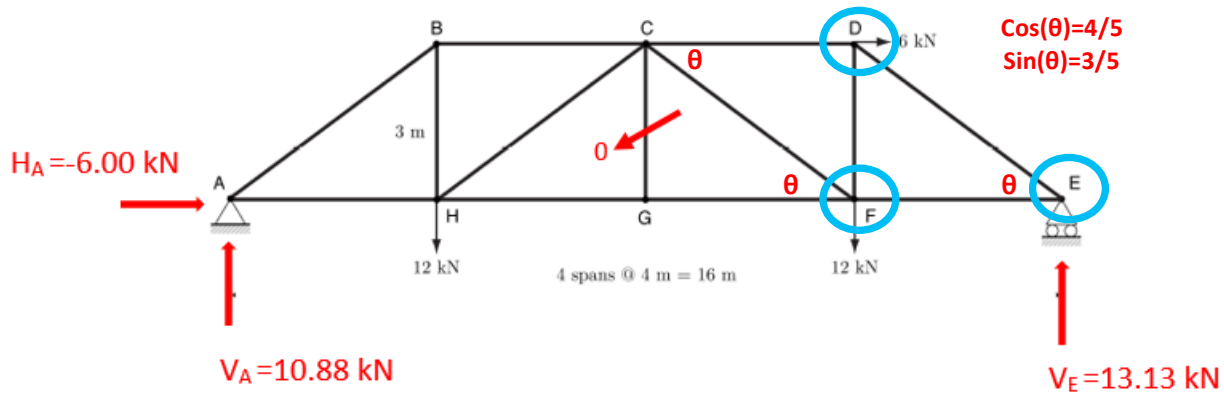
CG is a zero-force member

Bar CG satisfies the following zero-force conditions:

- *If three members form a joint, and two are co-linear, the third member must be a zero-force member provided no support reaction or applied loading at the joint.*

Part C

Method of Joints for bar forces CD, CF, FG



Joint E

$$\begin{aligned} 1^{\text{st}} \quad \Sigma F_y = 0 &\rightarrow F_{ED} \left(\frac{3}{5}\right) + V_E = 0 \\ &\Rightarrow F_{ED} = -21.875 \text{ kN} \end{aligned}$$

$$\begin{aligned} 2^{\text{nd}} \quad \Sigma F_x = 0 &\rightarrow -F_{EF} - F_{ED} \left(\frac{4}{5}\right) = 0 \\ &\Rightarrow F_{EF} = 17.50 \text{ kN} \end{aligned}$$

Joint F

$$\begin{aligned} 5^{\text{th}} \quad \Sigma F_y = 0 &\rightarrow F_{FD} + F_{FC} \left(\frac{3}{5}\right) - 12 = 0 \\ &\Rightarrow F_{FC} = -1.875 \text{ kN} \end{aligned}$$

$$6^{\text{th}} \quad \Sigma F_x =$$

Joint D

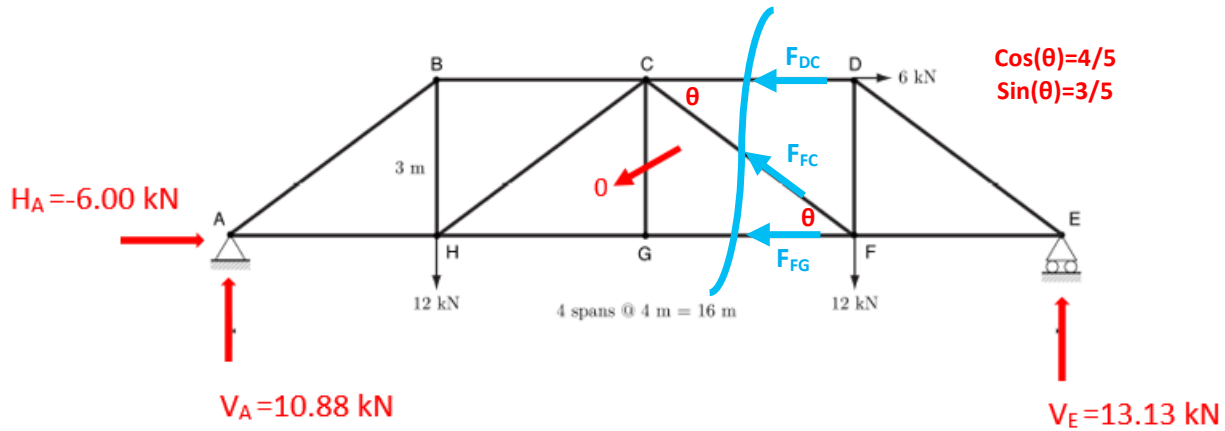
$$\begin{aligned} 3^{\text{rd}} \quad \Sigma F_x = 0 &\rightarrow F_{DE} \left(\frac{4}{5}\right) + 6 - F_{DC} = 0 \\ &\Rightarrow F_{DC} = -11.5 \text{ kN} \end{aligned}$$

$$\begin{aligned} 4^{\text{th}} \quad \Sigma F_y = 0 &\rightarrow -F_{DF} - F_{DE} \left(\frac{3}{5}\right) = 0 \\ &\Rightarrow F_{DF} = 13.125 \text{ kN} \end{aligned}$$

$$\begin{aligned} F_{FG} &= 19.00 \text{ kN (T)} \\ F_{FC} &= 1.88 \text{ kN (C)} \\ F_{DC} &= 11.50 \text{ kN (C)} \end{aligned}$$

Part D

Method of Sections for bar forces CD, CF, FG



$$\Sigma M_C = 0 \rightarrow -F_{FG}(3) - 12(4) + V_E(8) = 0 \rightarrow F_{FG} = 19.00 \text{ kN}$$

$$\Sigma F_y = 0 \rightarrow F_{FC} \left(\frac{3}{5} \right) - 12 + V_E = 0 \rightarrow F_{FC} = -1.875 \text{ kN}$$

$$\Sigma M_F = 0 \rightarrow -6(3) + V_E(4) + F_{DC}(3) = 0 \rightarrow F_{DC} = -11.50 \text{ kN}$$

$$F_{FG} = 19.00 \text{ kN (T)}$$

$$F_{FC} = 1.88 \text{ kN (C)}$$

$$F_{DC} = 11.50 \text{ kN (C)}$$