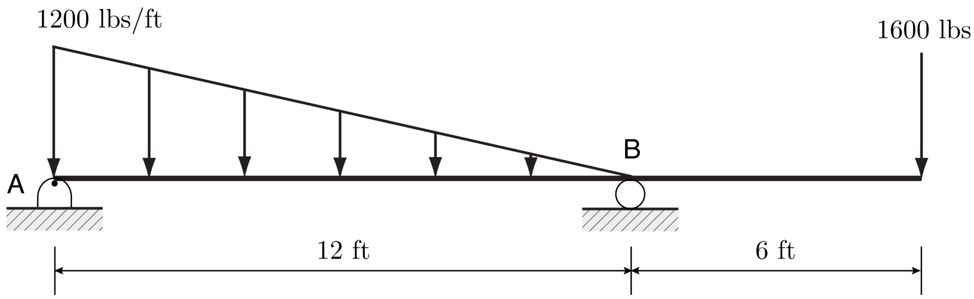
SOLUTION

CE 325 HW#2

1. For the determinate beam given below:

   a. (5 pts) Draw shear (V) and bending moment (M) diagrams and label with values at each transition point (beginning/end segments, maxima/minima, points of inflection). Show your sign conventions for the V and M diagrams.

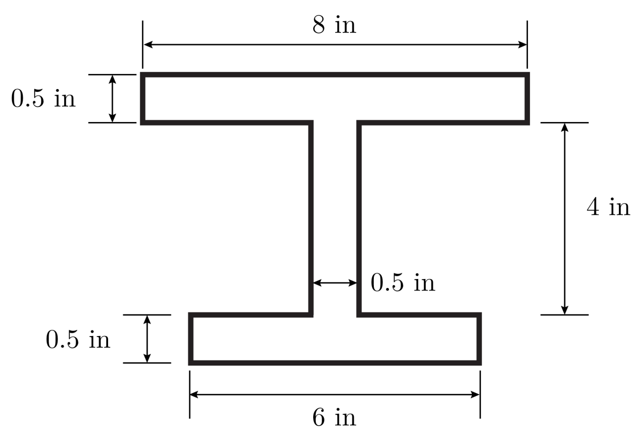


2. Given the cross-section of a beam below:

   a. (5 pts) Find the location of the Neutral Axis (N.A.) and moment of inertia about the N.A.

Given the max/min shear force (V_{max} = 4000 lbs, V_{min} = -3200 lbs) and bending moments (M_{max} = 7467 lbs·ft, M_{min} = -9600 lbs·ft), calculate the:

   b. (5 pts) Maximum tensile and compressive bending stresses (σ)
   c. (5 pts) Maximum shear stress (τ)



Problem 1

Executive Summary

Problem Statement

Draw V and M diagrams. Find maxima/minima and points of inflection.

Results

$$V_{max} = 4000\ lbs$$

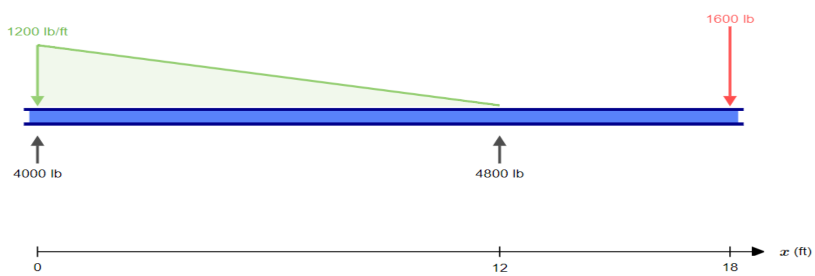
$$V_{min} = -3200\ lbs$$

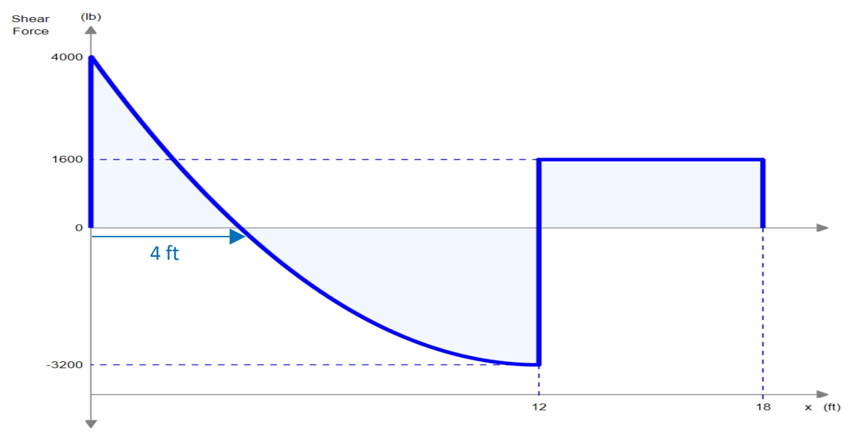
$$M_{max} = 7466.7\ lb * ft$$

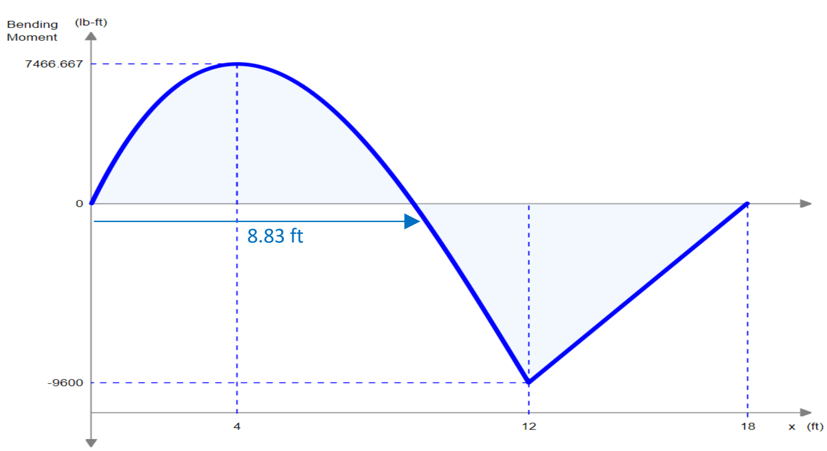
$$M_{min} = -9600\ lb * ft$$

$$Inflection\ point\ at$$

$$x = 8.83\ ft$$







Technical Summary

Step 1

Solve for resultant force and location.

$$F_R = \frac{1}{2} * 12ft * 1200\frac{lb}{ft}$$

$$=> F_R = 7200\ lbs$$

Load is triangular. F_R *located at one – third distance away from left support.*

$$=> F_R\ @\ x = 4ft$$

Step 2

Solve for reactions at supports.

$$\Sigma M_A = 0 \rightarrow -F_R(4) + B_y(12) - 1600(18) = 0$$

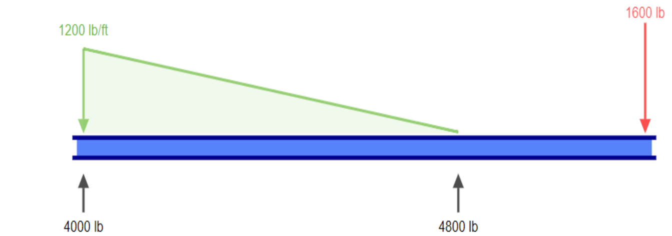
$$=> B_y = 4800\ lbs$$

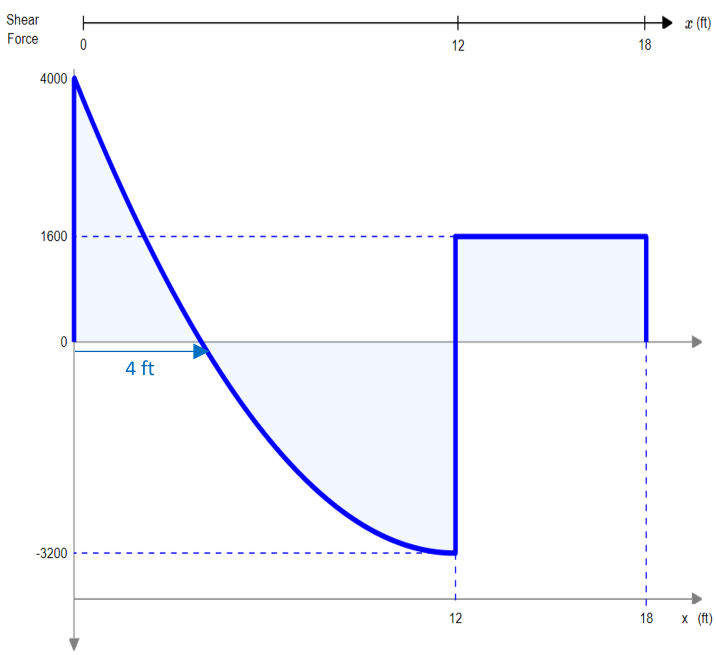
$$\Sigma M_B = 0 \rightarrow F_R(8) - A_y(12) - 1600(6) = 0$$

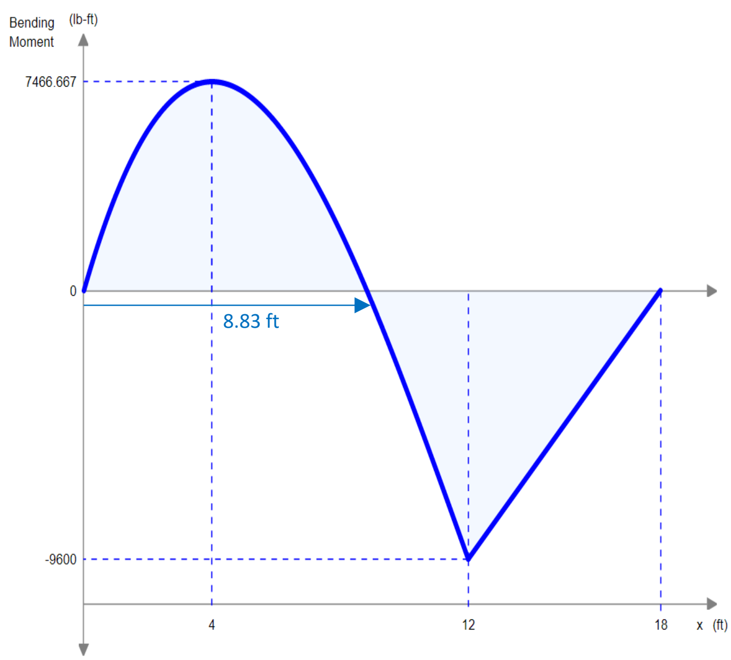
$$=> A_y = 4000\ lbs$$

Step 3

Draw V and M. Use integration to calculate V and M values.







$$\boldsymbol{q(x)_{0ft\to 12ft} = 100x - 1200}$$

$$V(x) = \int q(x)\,dx$$

$$V(x) = 50x^2 - 1200x + C$$

$$V(0) = 4000 \to C = 4000$$

$$\boldsymbol{V(x)_{0ft\to 12ft} = 50x^2 - 1200x + 4000}$$

$$Solve: V(x)_{0ft\to 12ft} = 0 \to x = 4ft$$

$$Moment\ curve\ reaches \max at\ 4ft$$

$$V(0)_{0ft\to 12ft} = 4000\ lbs$$

$$V(4)_{0ft\to 12ft} = 0\ lbs$$

$$V(12)_{0ft\to 12ft} = -3200\ lbs$$

$$V(x)_{12ft\to 18ft} = -3200 + 4800 = 1600\ lbs$$

$$M(x) = \int V(x)\,dx$$

$$M(x) = \frac{50}{3}x^3 - 600x^2 + 4000x + D$$

$$M(0) = 0 \to D = 0$$

$$\boldsymbol{M(x)_{0ft\to 12ft} = \frac{50}{3}x^3 - 600x^2 + 4000x}$$

$$Solve: M(x)_{0ft\to 12ft} = 0 \to x = 8.83ft$$

$$Inflection\ point\ at\ x = 8.83\ ft$$

$$M(0)_{0ft\to 12ft} = 0\ lb * ft$$

$$M(4)_{0ft\to 12ft} = 7466.7\ lb * ft$$

$$M(8.67)_{0ft\to 12ft} = 0\ lb * ft$$

$$M(12)_{0ft\to 12ft} = -9600\ lb * ft$$

$$M(18)_{12ft\to 18ft} = 0\ lb * ft$$

Step 4

Maxima and minima

$$V_{max} = 4000\ lbs$$

$$V_{min} = -3200\ lbs$$

$$M_{max} = 7466.7\ lb * ft$$

$$M_{min} = -9600\ lb * ft$$

Step 5

Inflection points

$$Inflection\ point\ at\ M(x) = 0$$

$$Inflection\ point\ at\ x = 8.83\ ft$$

Problem 2

Executive Summary

Problem Statement

Locate N.A.

Calculate max bending and shear stresses.

Results

$$\bar{y} = 2.75\ in.$$

$$I = 37.6874\ in^4$$

$$\sigma_{max}(T) = 6880\ psi$$

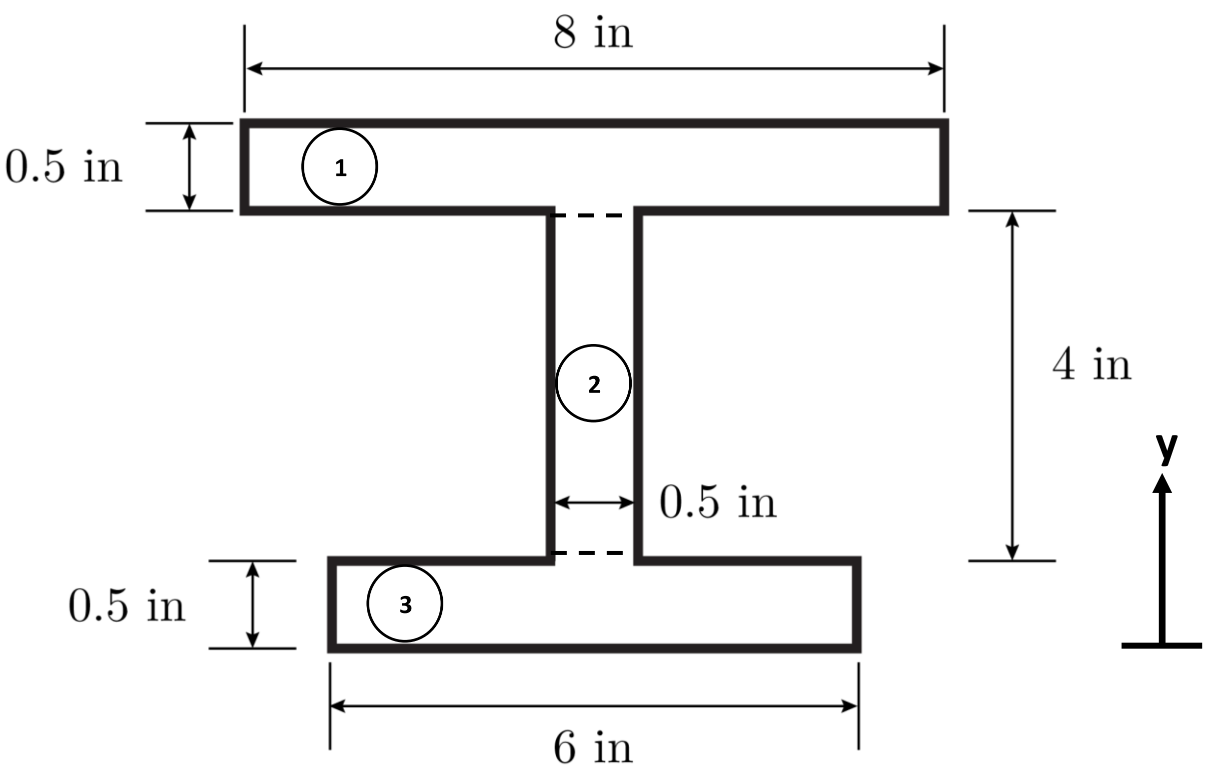
$$\sigma_{max}(C) = 8410\ psi$$

$$\tau_{xy\_max} = 1860\ psi$$

Technical Summary

Step 1

Locate N.A.



$$\bar{y} = \frac{\sum_i \bar{y}_i \bar{A}_i}{\sum_i \bar{A}_i}$$

$$\bar{y} = \frac{\left((4.75)(4) + (2.5)(2) + (0.25)(3)\right)}{(4 + 2 + 3)}$$

$$\bar{y} = 2.75\ in.$$

Step 2

Calculate Moment of Inertia

$\boldsymbol{Section}$ #	$\boldsymbol{I_{centroid}}$	$\boldsymbol{Area}$	$(\bar{y}_i - \bar{y})^2$	$\boldsymbol{I_i}$
1	$\left(\frac{1}{12}\right)(8)(0.5)^3$	4	$(4.75 - 2.75)^2$	16.0833
2	$\left(\frac{1}{12}\right)(0.5)(4)^3$	2	$(2.5 - 2.75)^2$	2.7917
3	$\left(\frac{1}{12}\right)(6)(0.5)^3$	3	$(0.25 - 2.75)^2$	18.8125
			$\boldsymbol{I =}$	$\boldsymbol{37.6874\ in^4}$

Step 3

Calculate Max Shear Stress

$$Q = \int y_i dA$$

$$Q(y = 0) = (4)(4.75 - 2.75) + (1.75 * 0.5)\left(2.75 + \frac{1.75}{2} - 2.75\right)$$

$$Q(y = 0) = 8.7656\ in^3$$

Note: shear stress is maximum at the N.A.

Note: y measured from N.A.

$$V_{max} = 4000\ lbs$$

$$V_{min} = -3200\ lbs$$

$$\tau_{xy} = \frac{VQ}{Ib}$$

$$\tau_{xy\_max} = \frac{(4000)(8.7656)}{(37.6874)(0.5)}$$

$$\tau_{xy\_max} = 1860\ psi$$

Step 4

Calculate Max Bending Stress

$$M_{max} = 7466.7\ lb * ft$$

$$M_{min} = -9600\ lb * ft$$

$$\sigma_x = -\frac{My}{I}$$

Note: normal bending stress is maximum at extreme fibers

Note: y measured from N.A.

$$-\frac{(7466.7)(+2.25)(12)}{(37.6874)} = -5350\ psi$$

$$-\frac{(7466.7)(-2.75)(12)}{(37.6874)} = 6540\ psi$$

$$-\frac{(-9600)(+2.25)(12)}{(37.6874)} = 6880\ psi$$

$$-\frac{(-9600)(-2.75)(12)}{(37.6874)} = -\ 8410\ psi$$

$$\sigma_{max}(T) = 6880\ psi$$

$$\sigma_{max}(C) = 8410\ psi$$