SOLUTION

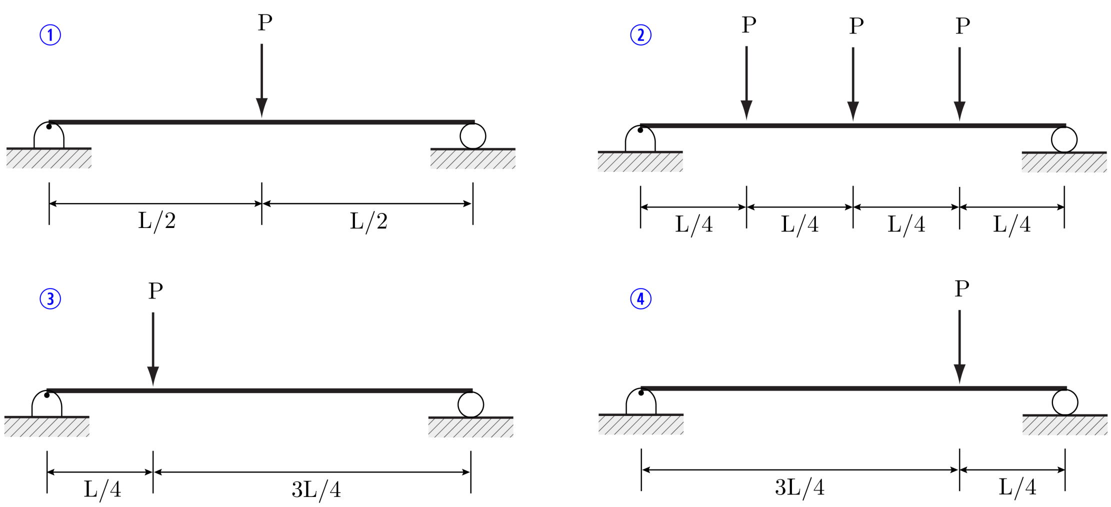
CE 325 Spring 2026 HW#3

Due Thursday, February 12, at the beginning of class

Note: This assignment utilizes a recently developed education research platform – *structural analysis integrated learning (SAIL)* – that is intended to help you develop a better "feel" for beam analysis and quickly identify/remedy calculation errors.

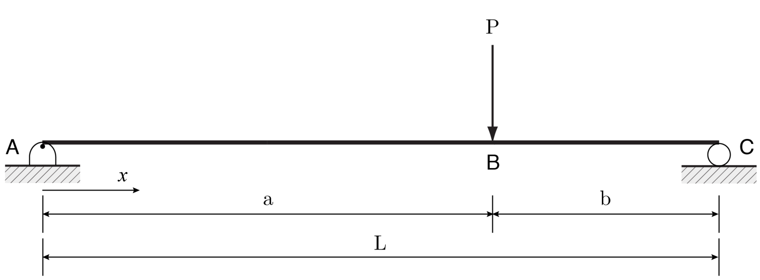
Training Exercises

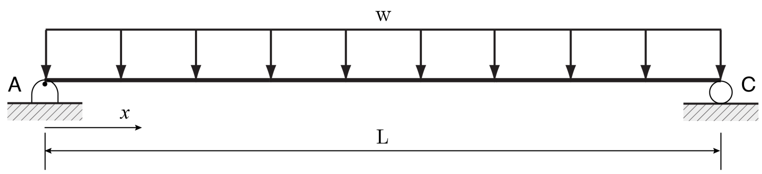
For the beams below with various loading and support conditions, use the *SAIL platform* to analyze the structural response (e.g., find reactions, deflections, shear force and bending moment diagrams). Feel free to create/model your own structures as well.



1. For the **concentrated force** and **uniformly distributed load** cases shown below:

   a. (5 pts) Using intuition developed in the *SAIL* training exercises above, try to sketch the *qualitative* **shear force** and **bending moment** diagrams without making any calculations.
   b. (5 pts) Use the *SAIL platform* to verify your results in Part (a).
   c. (5 pts) Using mechanics principles from class, determine the external reactions and draw the *quantitative* **shear force** and **bending moment** diagrams.
   d. (5 pts) Comment on your findings and why/why not you found the *SAIL platform* helpful to find the reactions and construct shear/moment diagrams.

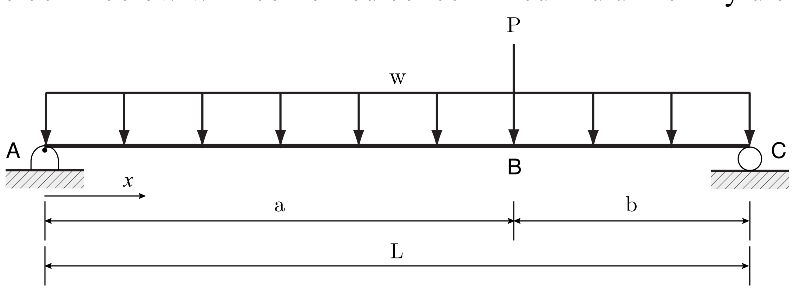




2. For the two loading scenarios in Problem 1 above:

   a. (5 pts) Using intuition developed in the *SAIL* training exercises above, try to sketch the *qualitative* **deflected shape** without making any calculations.
   b. (5 pts) Use the *SAIL platform* to verify your results in Part (a).
   c. (5 pts) Integrate the *Moment-Curvature* relation to obtain the equation(s) for the **rotation** and **deflection** as a function of x.
   d. (5 pts) Plot the deflected shape.
   e. (5 pts) Determine the locations and values of the **maximum displacement**.
   f. (5 pts) Comment on your findings and why/why not you found the *SAIL platform* helpful to determine the rotation and deflection behavior.

3. For the beam below with combined concentrated and uniformly distributed loads:



   a. (5 pts) Using knowledge and intuition developed in the prior exercises, try to sketch the *qualitative* **deflected shape** without making any calculations.
   b. (5 pts) Using the *Principal of Superposition*, determine the equation(s) for the **rotation** and **displacement** as a function of x.
   c. (5 pts) Find the **location** and **value** of the **maximum displacement** given:
      $P = 1000$ lbs, $w = 1$ k/ft, $a = 8$ ft., $b = 4$ ft., $E = 36e3$ ksi, $I = 12$ in.^{4}
   d. (5 pts) Use *SAIL platform* to verify your results in Part (c).
   e. (5 pts) Comment on your findings and why/why not you found the *SAIL platform* helpful to determine the structural response for the compound loaded beam.

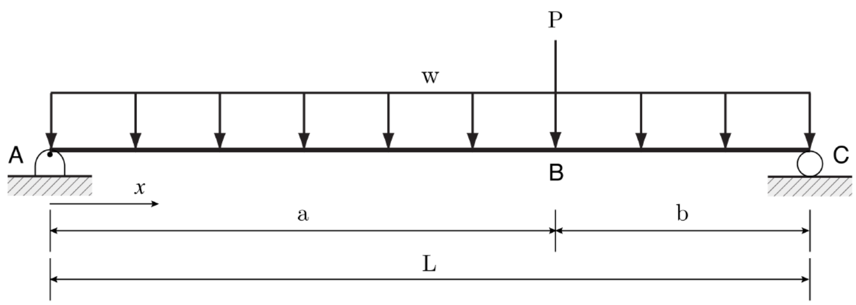
Problems 1-3

Executive Summary

Problem Statement

Using the Principal of Superposition, determine for the beam shown below:

a. The equations(s) for the displacement as a function of x.
b. The location and value of overall maximum displacement given:
   (P = 1000 lbs, w = 1 k/ft, a = 8 ft, b = 4 ft, E = 36 ksi, I = 12 in.^{4})



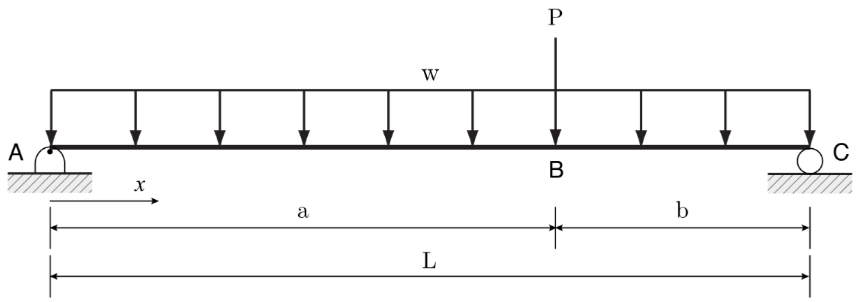
Results

$$v_{tot1}(x) = -\frac{wx}{24EI}(L^3 - 2L^2 + x^3) - \frac{Pbx}{6EIL}(L^2 - b^2 - x^2) \quad for\ 0 \le x \le a$$

$$v_{tot2}(x) = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3) - \frac{Pb}{6EIL}\left(\frac{L(-a+x)^3}{b} - x^3 + x(L^2 - b^2)\right) \quad for\ a \le x \le L$$

$$v_{max} = -1.20\ in\ @\ x = 6.06\ ft$$

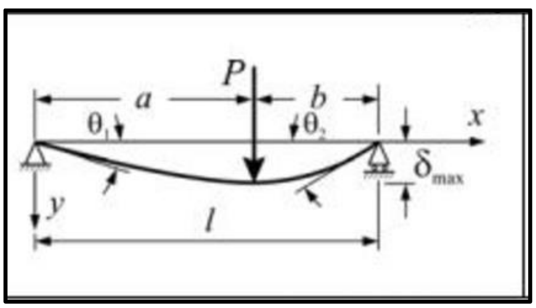
Technical Summary



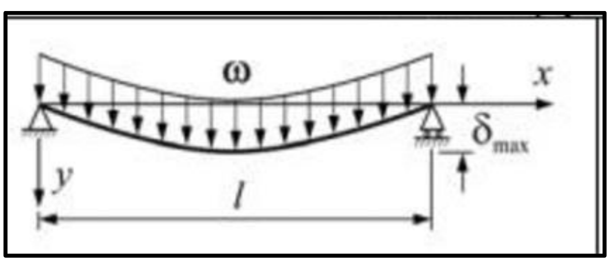
a. Equation(s) for the displacement as a function of x

The above loading condition can be represented by a linear combination of the two load cases below:

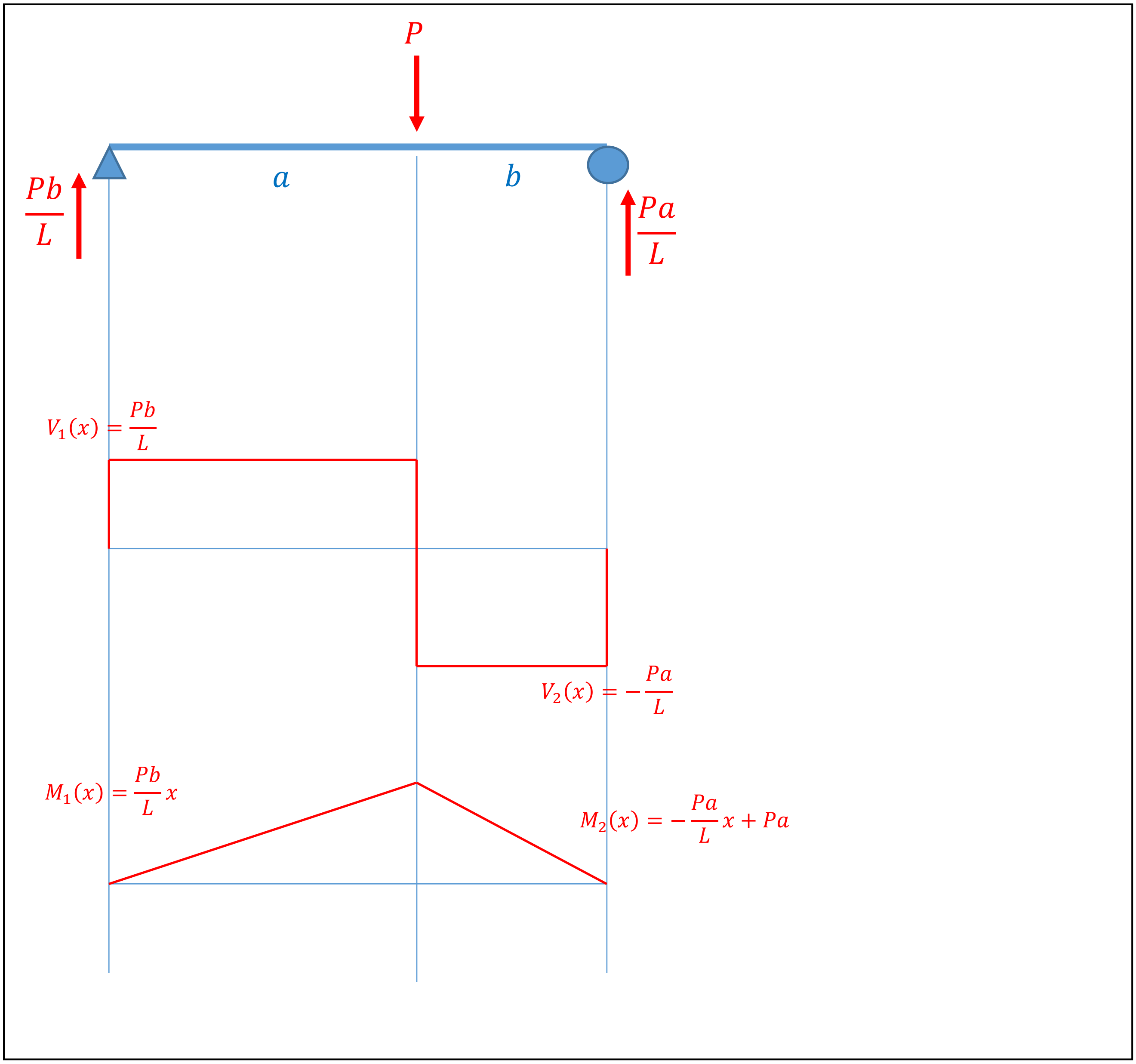
Simply supported beam – concentrated load



Simply supported beam – distributed load



Concentrated Load Case



Rotation and Displacement Equations with Constants

$$EIv_1'(x) = \frac{Pbx^2}{2L} + C_1 \quad for\ \ 0 \le x \le a$$

$$EIv_1(x) = \frac{Pbx^3}{6L} + C_1x + D_1 \quad for\ \ 0 \le x \le a$$

$$EIv_2'(x) = -\frac{Pax^2}{2L} + Pax + C_2 \quad for\ \ a \le x \le L$$

$$EIv_2(x) = -\frac{Pax^3}{6L} + \frac{Pax^2}{2} + C_2x + D_2 \quad for\ \ a \le x \le L$$

Boundary Conditions and Compatibility

$B.C.$ #1: $v_1(0) = 0$

$B.C.$ #2: $v_2(L) = 0$

$Compatibility$ #3: $v_1'(a) = v_2'(a)$

$Compatibility$ #4: $v_1(a) = v_2(a)$

$B.C.$ #1:

$$\boldsymbol{D_1 = 0}$$

$B.C.$ #2:

$$D_2 = -\frac{PaL^2}{3} - C_2L$$

$Compatibility$ #3:

$$\frac{Pba^2}{2L} + C_1 = -\frac{Pa^3}{2L} + Pa^2 + C_2$$

Substitute b = L-a

$$C_1 = \frac{Pa^2}{2} + C_2$$

Compatibility #4:

$$\frac{Pba^3}{6L} + C_1 a + D_1 = -\frac{Pa^4}{6L} + \frac{Pa^3}{2} + C_2 a + D_2$$

Substitute (above equation will be in terms of C_2)

$$D_1 = 0$$

$$D_2 = -\frac{PaL^2}{3} - C_2 L$$

$$C_1 = \frac{Pa^2}{2} + C_2$$

Then Solve

$$\boldsymbol{C_2 = -\frac{Pa^3}{6L} - \frac{PaL}{3}}$$

$$\boldsymbol{C_1 = \frac{Pa^2}{2} - \frac{Pa^3}{6L} - \frac{PaL}{3}}$$

B.C. #2:

$$D_2 = -\frac{PaL^2}{3} - C_2 L$$

Substitute C_2

$$\boldsymbol{D_2 = \frac{Pa^3}{6}}$$

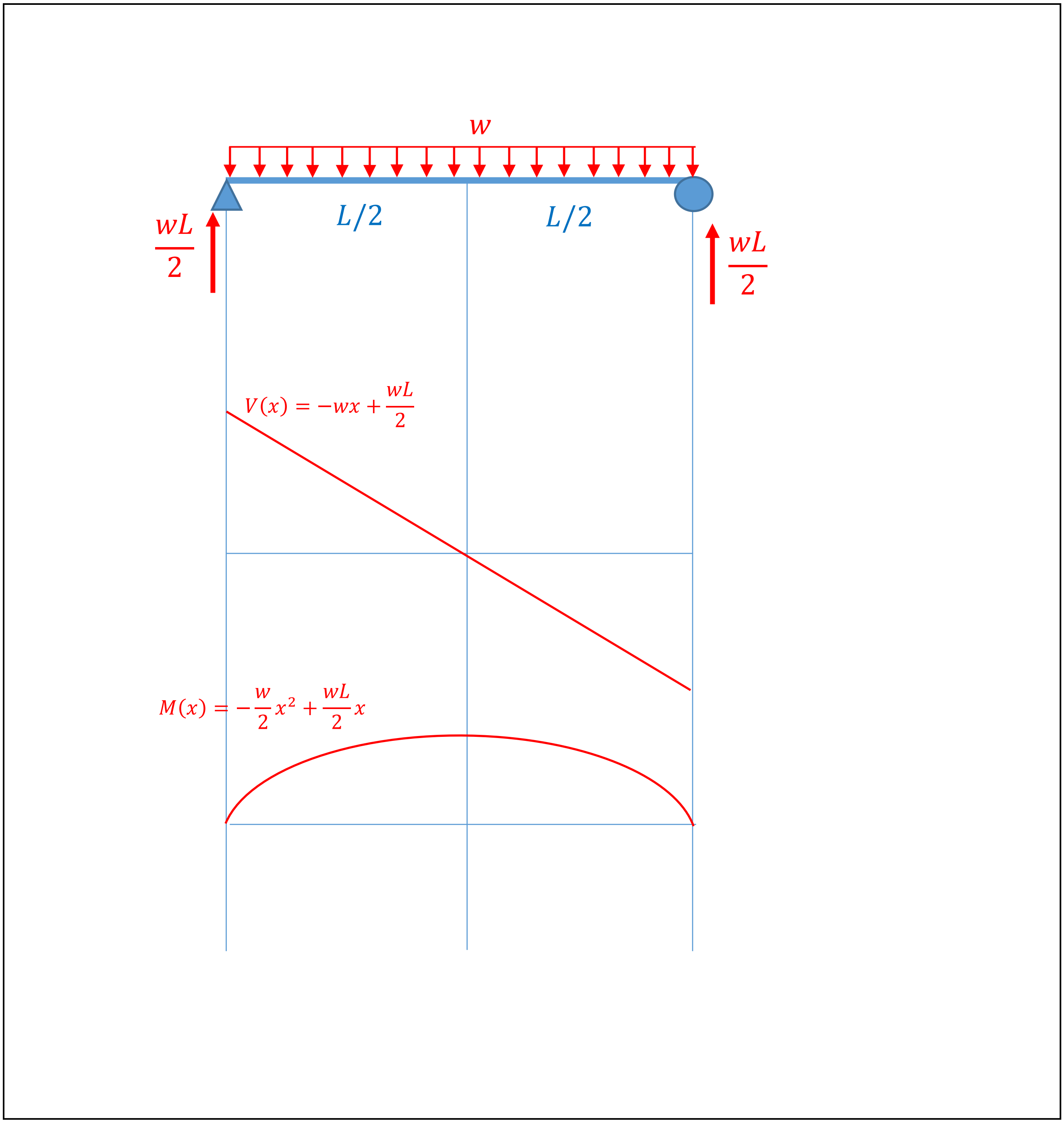
Simplify

**simplified form may vary*

$$\boldsymbol{v_1(x) = -\frac{Pbx}{6LEI}(L^2 - x^2 - b^2) \quad for\ 0 < x < a}$$

$$\boldsymbol{v_2(x) = -\frac{Pb}{6LEI}\left(\frac{L}{b}(x-a)^3 + (L^2 - b^2)x - x^3\right) \quad for\ a < x < L}$$

Distributed Load Case



Rotation and Displacement Equations with Constants

$$EIv'(x) = -\frac{wx^3}{6} + \frac{wLx^2}{4} + C$$

$$EIv(x) = -\frac{wx^4}{24} + \frac{wLx^3}{12} + Cx + D$$

Boundary Conditions and Compatibility

#1: $v(0) = 0$

#2: $v(L) = 0$

#1:

$$\boldsymbol{D = 0}$$

#2:

$$\boldsymbol{C = -\frac{wL^3}{24}}$$

Simplify

**simplified form may vary*

$$\boldsymbol{v_w(x) = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3) \quad for \;\; 0 \le x \le L}$$

Superposition

<u>Concentrated Load:</u>

$$v_1(x) = -\frac{Pbx}{6LEI}(L^2 - x^2 - b^2) \quad for\ 0 \le x \le a$$

$$v_2(x) = -\frac{Pb}{6LEI}\left(\frac{L}{b}(x-a)^3 + (L^2 - b^2)x - x^3\right) \quad for\ a \le x \le L$$

<u>Distributed Load:</u>

$$v(x) = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3) \quad for\ 0 \le x \le L$$

<u>Superposition:</u>

To find total displacement equation(s), add uniform load equation to $v_1(x)$ and $v_2(x)$

$$\boldsymbol{v_{tot1}(x) = -\frac{wx}{24EI}(L^3 - 2L^2 + x^3) - \frac{Pbx}{6EIL}(L^2 - b^2 - x^2) \quad for\ 0 \le x \le a}$$

$$\boldsymbol{v_{tot2}(x) = -\frac{wx}{24EI}(L^3 - 2Lx^2 + x^3) - \frac{Pb}{6EIL}\left(\frac{L(-a+x)^3}{b} - x^3 + x(L^2 - b^2)\right) \quad for\ a \le x \le L}$$

b. Location and value of overall max displacement

Check maxima/minima within the bounds of the displacement equations

$v'_{tot1}(x) = 0 \rightarrow x = (-4.49, 6.06, 17.44)\ ft \quad for\ 0\ ft < x < 8\ ft$

Max at x = 6.06 ft

$v_{tot1}(x = 6.06ft) = -1.20\ in$

$v'_{tot2}(x) = 0 \rightarrow x = (-6.60, 6.14, 16.45)\ ft \quad for\ 8\ ft < x < 12\ ft$

There are no maxima/minima within the range of deflection equation 2

Check boundary locations of the displacement equations

$v_{tot1}(x = 0\ ft) = 0$

$v_{tot2}(x = 12\ ft) = 0$

$v_{tot1}(x = 8\ ft) = v_{tot2}(x = 8\ ft) = -1.05\ in$

Max displacement and location

$\boldsymbol{v_{max} = -1.20\ in\ @\ x = 6.06\ ft}$