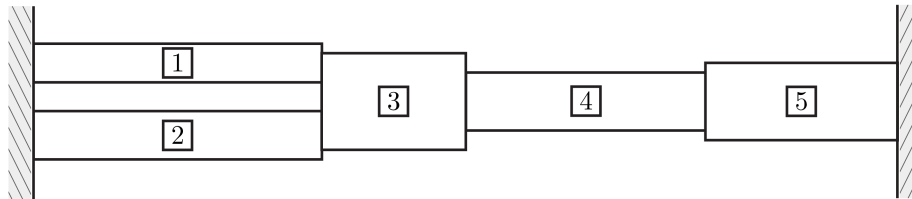


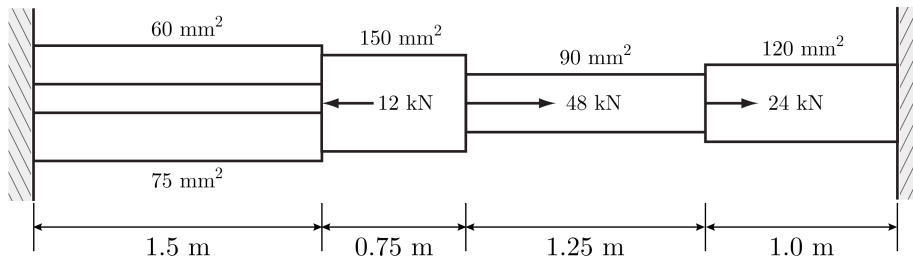
CE 325 Spring 2026 HW#4

Due Thursday, February 19, at the beginning of class

1. Analyze the uniaxial structure shown below using the Matrix Displacement Method:



- a. (2.5 pts) Use the notation outlined in class to label all: joints, degrees of freedom, loads, and reactions.
- b. (5 pts) Use the definition of structural level stiffness (S_{ij}) and superposition to develop the structural force-displacement relationship: $\{P\} = [S]\{d\}$.
- c. (5 pts) Use the definition of member level stiffness (k_{ij}) and superposition to develop the member force-displacement relationship: $\{Q\} = [k]\{u\}$.
- d. (5 pts) Use the Rigorous Method to assemble $[S]$ in terms of the k_{ij} 's for each member.
- e. (2.5 pts) Use the Code Number Method to assemble $[S]$ in terms of the k_{ij} 's for each member.
- f. (5 pts) Given the applied loads, member geometry, and material properties below: determine displacements, member forces (indicate tension or compression), and support reactions.



$E = 20 \text{ GPa}$ for all members

Problem 1

Executive Summary

Problem Statement

- Label joints, DOFs, loads, and reactions
- Develop the structural force-displacement relationship: $\{P\} = [S]\{d\}$
- Develop the member force-displacement relationship: $\{Q\} = [k]\{u\}$
- Assemble $[S]$ using the Rigorous Method
- Assemble $[S]$ using the Code Number Method
- Determine displacements, member forces, and support reactions

Results

$$[S] = \begin{bmatrix} (k_{22}^1 + k_{22}^2 + k_{11}^3) & k_{12}^3 & 0 \\ k_{21}^3 & k_{22}^3 + k_{11}^4 & k_{12}^4 \\ 0 & k_{21}^4 & k_{22}^4 + k_{11}^5 \end{bmatrix}$$

$$\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 13.623 \\ 22.754 \\ 14.783 \end{Bmatrix} mm$$

$$Q^1 = 10.9 \text{ kN (T)}$$

$$Q^2 = 13.6 \text{ kN (T)}$$

$$Q^3 = 36.5 \text{ kN (T)}$$

$$Q^4 = 11.5 \text{ kN (C)}$$

$$Q^5 = 35.5 \text{ kN (C)}$$

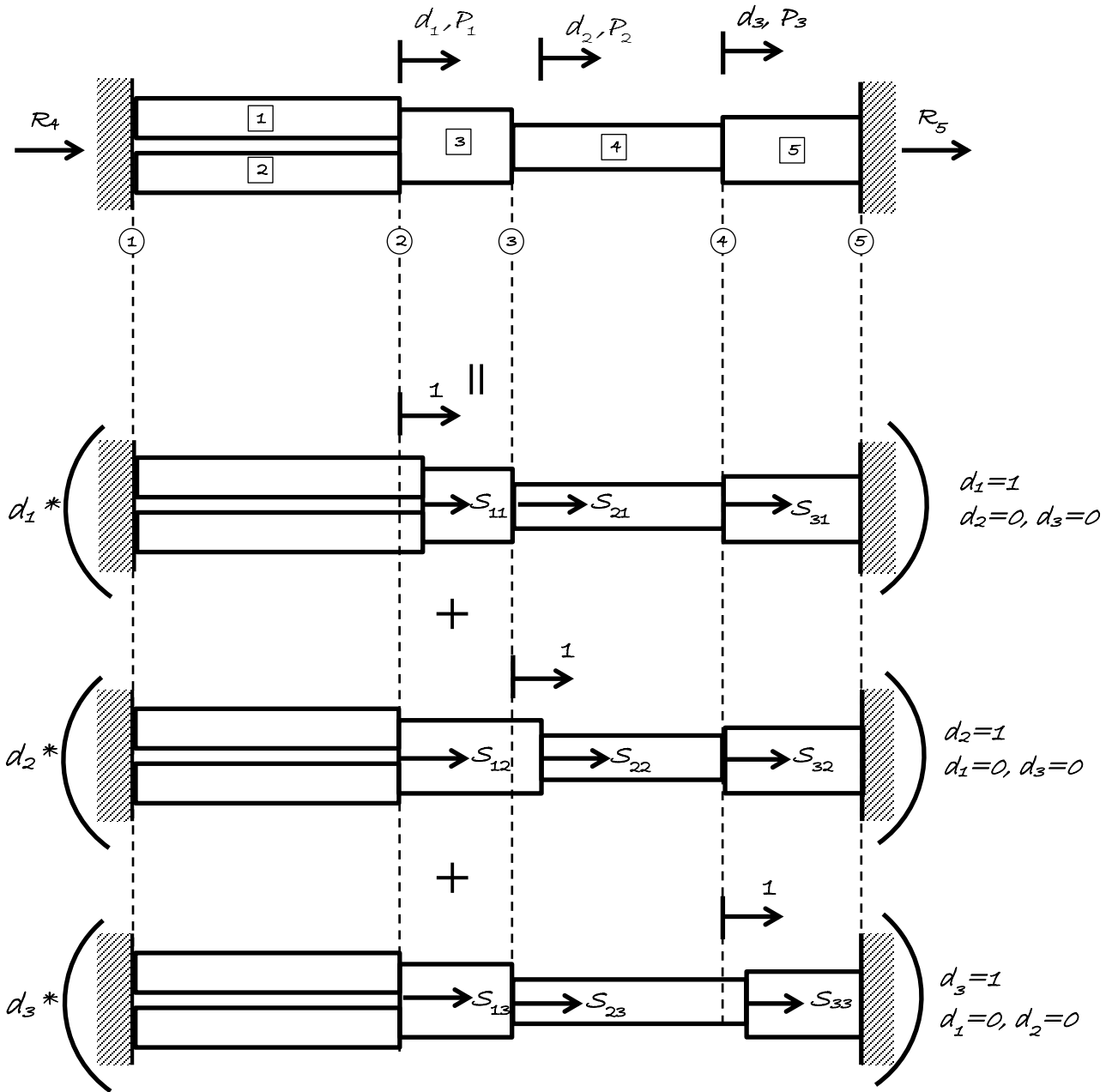
$$R_4 = -24.5 \text{ kN}$$

$$R_5 = -35.5 \text{ kN}$$

Technical Summary

a. Label joints, DOFs, loads, and reactions

Structure Level Superposition



b. Develop the structural force-displacement relationship: $\{P\} = [S]\{d\}$

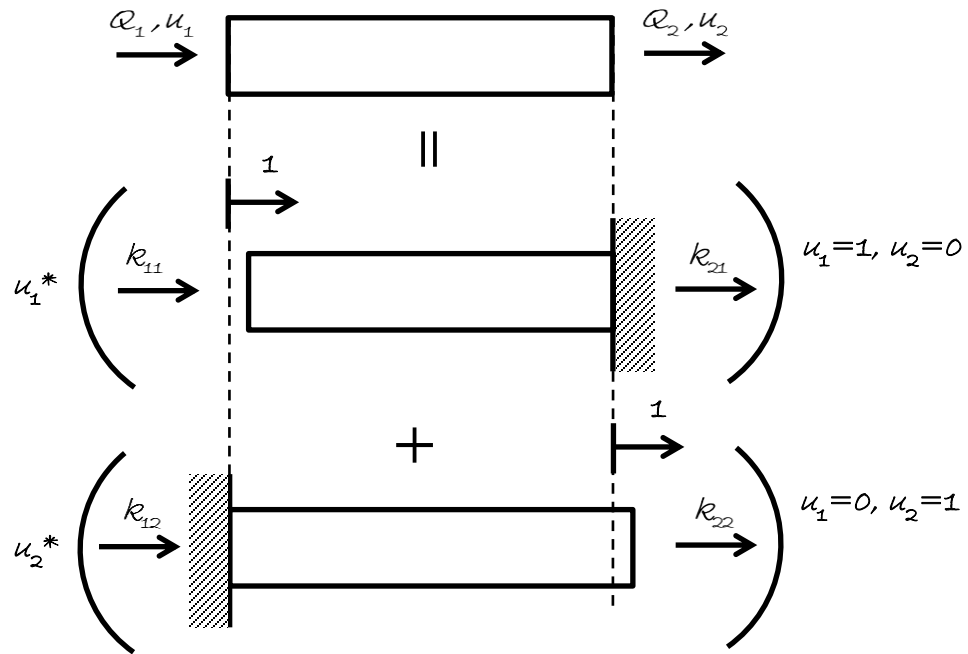
$$P_1 = S_{11}d_1 + S_{12}d_2 + S_{13}d_3$$

$$P_2 = S_{21}d_1 + S_{22}d_2 + S_{23}d_3 \Rightarrow \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} \text{ or } \{P\} = [S]\{d\}$$

$$P_3 = S_{31}d_1 + S_{32}d_2 + S_{33}d_3$$

c. Develop the member force-displacement relationship: $\{Q\} = [k]\{u\}$

Member Level Superposition

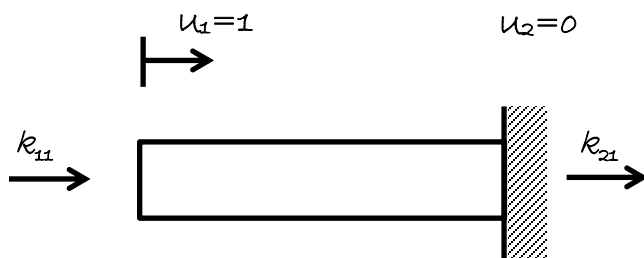


Member-Level Superposition

$$Q_1 = k_{11}u_1 + k_{12}u_2$$

$$Q_2 = k_{21}u_1 + k_{22}u_2$$

$$\Rightarrow \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} \text{ or } \{Q\} = [k]\{u\}$$



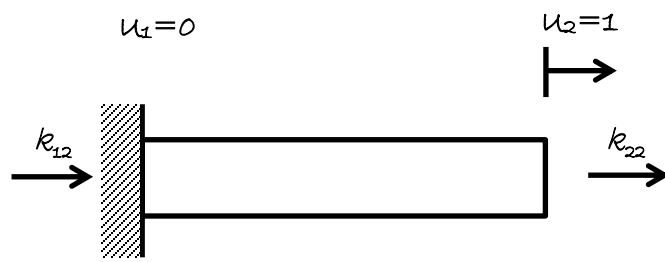
$$\text{Force} = \text{stiffness} * \text{displacement} \rightarrow P = \left(\frac{EA}{L}\right) \delta \rightarrow \delta = u_1 = 1$$

$$k_{11} = +\frac{EA}{L}$$

$$\sum F_x = 0 \rightarrow k_{11} + k_{21} = 0$$

$$k_{21} = -k_{11}$$

$$k_{21} = -\frac{EA}{L}$$



$$k_{22} = +\frac{EA}{L}$$

$$\sum F_x = 0 \rightarrow k_{12} + k_{22} = 0$$

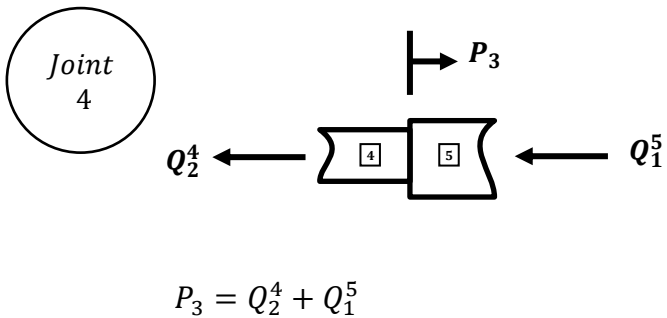
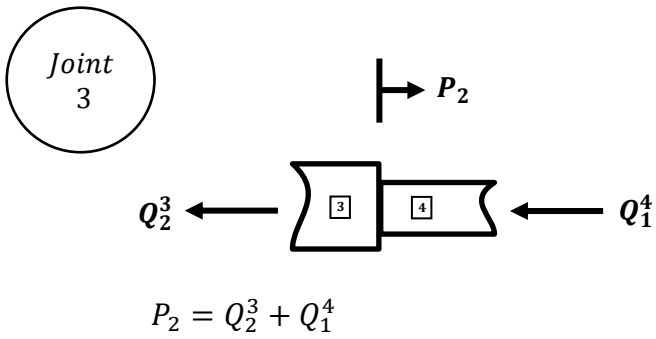
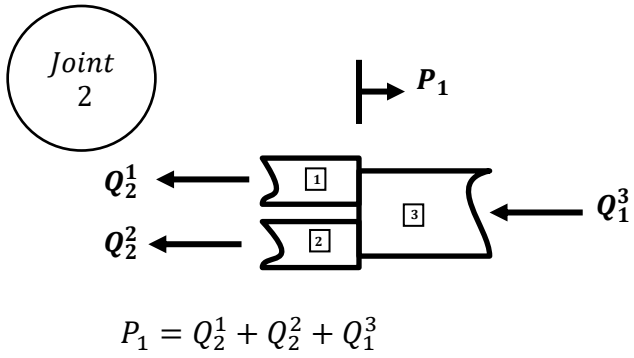
$$k_{12} = -k_{22}$$

$$k_{12} = -\frac{EA}{L}$$

$$[k] = \frac{EA}{L} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

d. Assemble [S] using the Rigorous Method

$\sum F_x$ at each joint



$\Sigma F_x = 0$ at each joint

$$P_1 = Q_2^1 + Q_2^2 + Q_1^3$$

$$P_2 = Q_2^3 + Q_1^4$$

$$P_3 = Q_2^4 + Q_1^5$$

$$P_1 = (k_{21}^1 u_1^1 + k_{22}^1 u_2^1) + (k_{21}^2 u_1^2 + k_{22}^2 u_2^2) + (k_{11}^3 u_1^3 + k_{12}^3 u_2^3)$$

$$P_2 = (k_{21}^3 u_1^3 + k_{22}^3 u_2^3) + (k_{11}^4 u_1^4 + k_{12}^4 u_2^4)$$

$$P_3 = k_{21}^4 u_1^4 + k_{22}^4 u_2^4 + k_{11}^5 u_1^5 + k_{12}^5 u_2^5$$

$$P_1 = (k_{22}^1 + k_{22}^2 + k_{11}^3) d_1 + k_{12}^3 d_2$$

$$P_2 = k_{21}^3 d_1 + (k_{22}^3 + k_{11}^4) d_2 + k_{12}^4 d_3$$

$$P_3 = k_{21}^4 d_2 + (k_{22}^4 + k_{11}^5) d_3$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} (k_{22}^1 + k_{22}^2 + k_{11}^3) & k_{12}^3 & 0 \\ k_{21}^3 & k_{22}^3 + k_{11}^4 & k_{12}^4 \\ 0 & k_{21}^4 & k_{22}^4 + k_{11}^5 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

e. Assemble [S] using the Code Number Method

Member #	Code Number	
Member 1	4	1
Member 2	4	1
Member 3	1	2
Member 4	2	3
Member 5	3	5

Members 1&2	
4	1
4	$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$

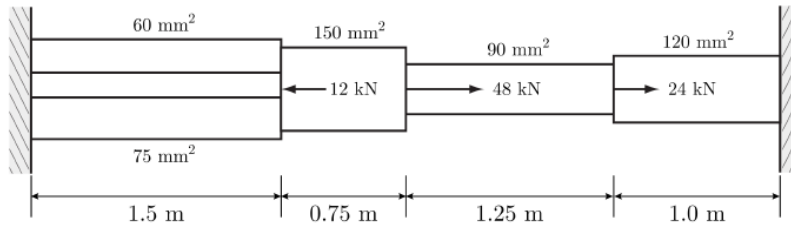
Member 3	
1	2
1	$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$

Members 4	
2	3
2	$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$

Members 5	
3	5
3	$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$

$$[S] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} (k_{22}^1 + k_{22}^2 + k_{11}^3) & k_{12}^3 & 0 \\ k_{21}^3 & k_{22}^3 + k_{11}^4 & k_{12}^4 \\ 0 & k_{21}^4 & k_{22}^4 + k_{11}^5 \end{bmatrix} \end{matrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

f. Determine displacements, member forces, and reactions



$E = 20 \text{ GPa}$ for all members

Calculate Member Level Stiffness

$$[k] = \frac{EA}{L} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

Member 1

$$[k] = \frac{(20e3 \frac{N}{mm^2})(60 \text{ mm}^2)}{(1500 \text{ mm})} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$[k] = \left(800 \frac{N}{mm}\right) \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

Member 2

$$[k] = \frac{(20e3 \frac{N}{mm^2})(75 \text{ mm}^2)}{(1500 \text{ mm})} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$[k] = \left(1000 \frac{N}{mm}\right) \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

Member 3

$$[k] = \frac{(20e3 \frac{N}{mm^2})(150 \text{ mm}^2)}{(750 \text{ mm})} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$[k] = \left(4000 \frac{N}{mm}\right) \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

Member 4

$$[k] = \frac{(20e3 \frac{N}{mm^2})(90 \text{ mm}^2)}{(1250 \text{ mm})} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$[k] = \left(1440 \frac{N}{mm}\right) \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

Member 5

$$[k] = \frac{(20e3 \frac{N}{mm^2})(120 \text{ mm}^2)}{(1000 \text{ mm})} \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

$$[k] = \left(2400 \frac{N}{mm}\right) \begin{bmatrix} +1 & -1 \\ -1 & +1 \end{bmatrix}$$

Assemble and Solve for d

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{bmatrix} (k_{22}^1 + k_{22}^2 + k_{11}^3) & k_{12}^3 & 0 \\ k_{21}^3 & k_{22}^3 + k_{11}^4 & k_{12}^4 \\ 0 & k_{21}^4 & k_{22}^4 + k_{11}^5 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

$$\begin{Bmatrix} -12000 \\ 48000 \\ 24000 \end{Bmatrix} = \begin{bmatrix} 5800 & -4000 & 0 \\ -4000 & 5440 & -1440 \\ 0 & -1440 & 3840 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

$$\begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix} = \begin{Bmatrix} 13.623 \\ 22.754 \\ 14.783 \end{Bmatrix} \text{ mm}$$

Compatibility

$$\{u\}^1 = \begin{Bmatrix} u_1^1 \\ u_2^1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ d_1 \end{Bmatrix} \quad \{u\}^4 = \begin{Bmatrix} u_1^4 \\ u_2^4 \end{Bmatrix} = \begin{Bmatrix} d_2 \\ d_3 \end{Bmatrix}$$

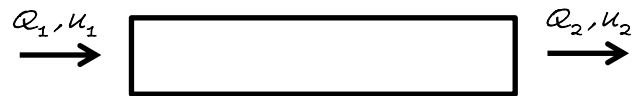
$$\{u\}^2 = \begin{Bmatrix} u_1^2 \\ u_2^2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ d_1 \end{Bmatrix} \quad \{u\}^5 = \begin{Bmatrix} u_1^5 \\ u_2^5 \end{Bmatrix} = \begin{Bmatrix} d_3 \\ 0 \end{Bmatrix}$$

$$\{u\}^3 = \begin{Bmatrix} u_1^3 \\ u_2^3 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix}$$

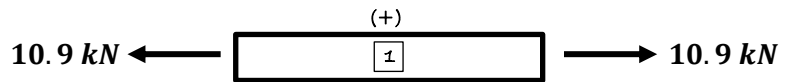
Calculate Member Forces

$$\{Q\} = [k]\{u\}$$

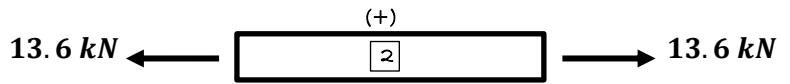
General Uniaxial Element:



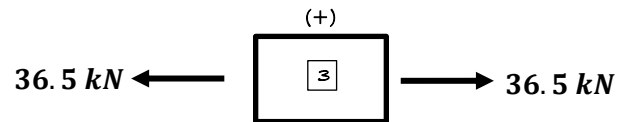
$$\{Q\}^1 = \begin{Bmatrix} -10.9 \\ 10.9 \end{Bmatrix} kN \rightarrow 10.9 kN (T)$$



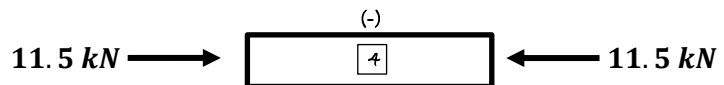
$$\{Q\}^2 = \begin{Bmatrix} -13.6 \\ 13.6 \end{Bmatrix} kN \rightarrow 13.6 kN (T)$$



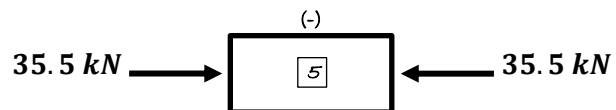
$$\{Q\}^3 = \begin{Bmatrix} -36.5 \\ 36.5 \end{Bmatrix} kN \rightarrow 36.5 kN (T)$$



$$\{Q\}^4 = \begin{Bmatrix} 11.5 \\ -11.5 \end{Bmatrix} kN \rightarrow 11.5 kN (C)$$

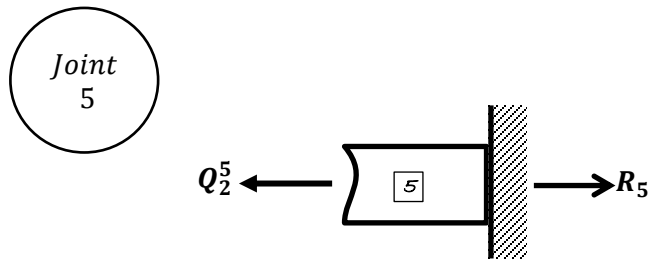
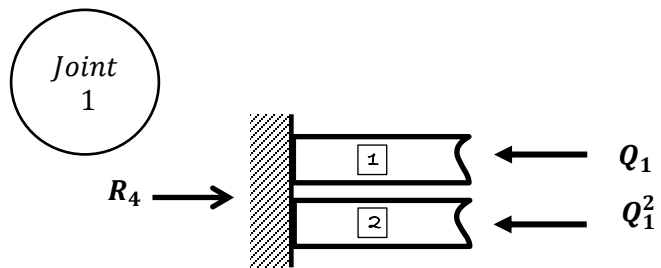


$$\{Q\}^5 = \begin{Bmatrix} 35.5 \\ -35.5 \end{Bmatrix} kN \rightarrow 35.5 kN (C)$$



Reactions

$\sum F_x = 0$ at each reaction joint



$$R_4 = Q_1^1 + Q_1^2 = -10.9 \text{ kN} - 13.6 \text{ kN}$$

$$R_5 = Q_2^5 = -35.5 \text{ kN}$$

$$\boxed{R_4 = -24.5 \text{ kN}}$$

$$\boxed{R_5 = -35.5 \text{ kN}}$$