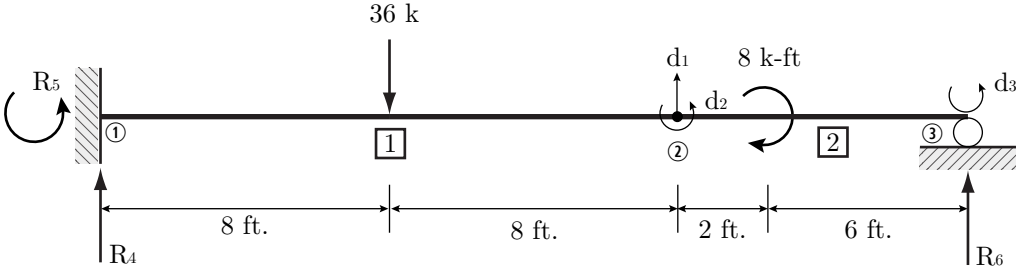
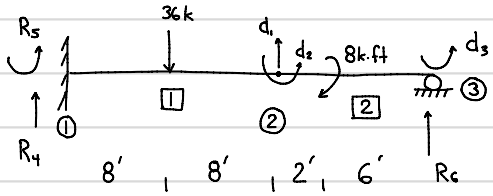


E = 4000 ksi; I = 1500 in.^{4} for all members

Beam Example

1.



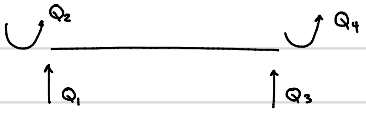
$E = 4{,}000$ ksi , $I = 1500$ in^4 for all members

$EI = 6e6$ k·in^2

2. Create $\{P\}$

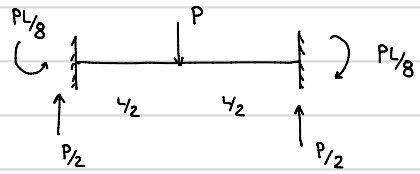
$$\{P\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

NO loads applied at DOFs



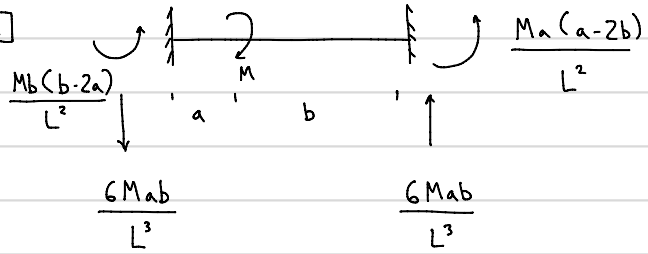
3. Determine $\{Qf\}$, $[k]$

1



$$\{Qf\}^1 = \begin{Bmatrix} P/2 \\ PL/8 \\ P/2 \\ -PL/8 \end{Bmatrix} = \begin{Bmatrix} 18 \\ 864 \\ 18 \\ -864 \end{Bmatrix} \begin{matrix} \text{k} \\ \text{k·in} \\ \text{k} \\ \text{k·in} \end{matrix}$$

2



$$\{Qf\}^2 = \begin{Bmatrix} -\frac{6Mab}{L^3} \\ \frac{Mb(b-2a)}{L^2} \\ \frac{6Mab}{L^3} \\ \frac{Ma(a-2b)}{L^2} \end{Bmatrix} = \begin{Bmatrix} -1.125 \\ 18 \\ 1.125 \\ -30 \end{Bmatrix} \begin{matrix} \text{k} \\ \text{k·in} \\ \text{k} \\ \text{k·in} \end{matrix}$$

$$[k] = \frac{EI}{L^3} \begin{bmatrix} (12) & 6L & -12 & 6L \\ (6L) & (4L^2) & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & (2L^2) & -6L & 4L^2 \end{bmatrix} \begin{matrix} u_1 & \Delta & \text{in.} \\ u_2 & \theta & \text{rad} \\ u_3 & \Delta & \text{in.} \\ u_4 & \theta & \text{rad.} \end{matrix}$$

$$\{Q\} = \{Qf\} + [k]\{u\}$$

1 $L = 16 \times 12 = 192$ in.

$EI = 6e6$ k·in^2

$\frac{12EI}{L^3} = 10.173$ k/in.

$\frac{4EI}{L} = 1.25e5$ k·in

$\frac{6EI}{L^2} = 976.56$ k

$\frac{2EI}{L} = 0.625e5$ k·in

[2] $L = 8 \times 12 = 96''$ $\quad EI = 6e6 \ k \cdot in^2$

$\frac{12EI}{L^3} = 81.38 \ k/in$ $\qquad \frac{4EI}{L} = 2.5e5 \ k \cdot in$

$\frac{6EI}{L^2} = 3906.2 \ k$ $\qquad \frac{2EI}{L} = 1.25e5 \ k \cdot in$

4. Assembly (Code #)

member # \ code #	u_1, Q_1	u_2, Q_2	u_3, Q_3	u_4, Q_4
[1]	4	5	1	2
[2]	1	2	6	3

$$\{Pf\} = \begin{Bmatrix} Pf_1 \\ Pf_2 \\ Pf_3 \end{Bmatrix} = \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \begin{Bmatrix} Qf_3^{'} + Qf_1^{2} \\ Qf_4^{'} + Qf_2^{2} \\ Qf_4^{2} \end{Bmatrix} \qquad [S] = \begin{matrix} \\ 1 \\ 2 \\ 3 \end{matrix} \begin{matrix} \begin{matrix} 1 & \quad & 2 & \quad & 3 \end{matrix} \\ \begin{bmatrix} k_{33}^{'} + k_{11}^{2} & k_{34}^{'} + k_{12}^{2} & k_{14}^{2} \\ k_{43}^{'} + k_{21}^{2} & k_{44}^{'} + k_{22}^{2} & k_{24}^{2} \\ k_{41}^{2} & k_{42}^{2} & k_{44}^{2} \end{bmatrix} \end{matrix}$$

$$\{Pf\} = \begin{Bmatrix} 16.875 \\ -846 \\ -30 \end{Bmatrix} \begin{matrix} k \\ k \cdot in \\ k \cdot in \end{matrix} \qquad [S] = \begin{bmatrix} 91.56 & 2929.7 & 3906.2 \\ 2929.7 & 3.75e5 & 1.25e5 \\ 3906.2 & 1.25e5 & 2.5e5 \end{bmatrix}$$

5. Solve for $\{d\}$

$$\{P - Pf\} = [S]\{d\} \quad \therefore \quad \{d\} = [S]^{-1}\{P - Pf\} \qquad \{d\} = \begin{Bmatrix} -0.726 \\ 0.00493 \\ 0.009 \end{Bmatrix} \begin{matrix} in. \\ rad \\ rad \end{matrix}$$

6. Compute $\{Q\}$ $\qquad \{Q\} = \{Qf\} + [k]\{u\}$

known known known — compatibility

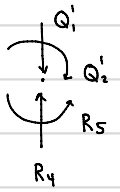
$$\{u\}^1 = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ d_1 \\ d_2 \end{Bmatrix} \begin{matrix} \text{fixed B.C.} \\ \text{fixed B.C.} \\ -0.726 \\ 0.00493 \end{matrix} \qquad \{u\}^2 = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \\ 0 \\ d_3 \end{Bmatrix} \begin{matrix} -0.726 \\ 0.00493 \\ \text{roller B.C.} \\ 0.009 \end{matrix}$$

$$\{Q\}^1 = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \begin{Bmatrix} 30.198 \\ 1881 \\ 5.8021 \\ 461 \end{Bmatrix} \begin{matrix} k \\ k\cdot in \\ k \\ k\cdot in \end{matrix}$$

$$\{Q\}^2 = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \begin{Bmatrix} -5.8021 \\ -461 \\ 5.8021 \\ -2.34\,e{-13} \end{Bmatrix} \begin{matrix} k \\ k\cdot in \\ k \\ k\cdot in \end{matrix} \approx 0$$

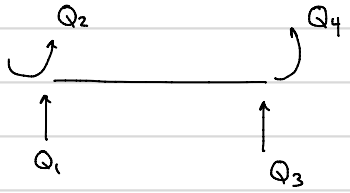
7. Reactions

joint ①

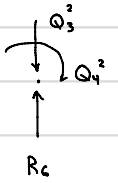


$\Sigma F_y = 0 \quad R_4 = Q'_1 = 30.198\ k$

$\Sigma M = 0 \quad R_S = Q'_2 = 156.75\ k\cdot ft$



joint ③

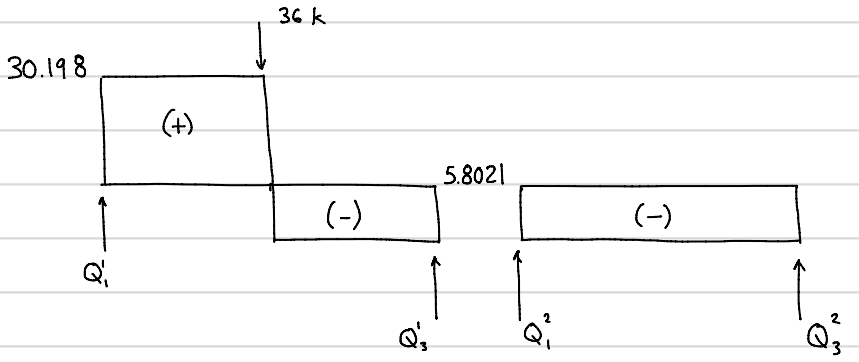


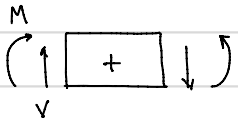
$\Sigma F_y = 0 \quad R_C = Q_3^2 = 5.8021\ k$

$\Sigma M = 0 \quad Q_4^2 = 0 \quad (-2.34\,e{-13})$ numerical error

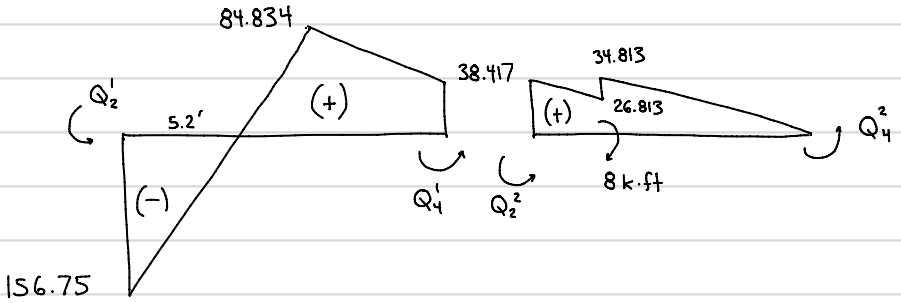
* Quick check : Verify overall equilibrium *

8. Draw shear force and bending moment diagrams





V(x) - k



M(x) - k·ft

9. Calculate Stresses - Normal/bending: $\sigma_b = \frac{-My}{I}$ Shear: $\tau = \frac{VQ}{Ib}$