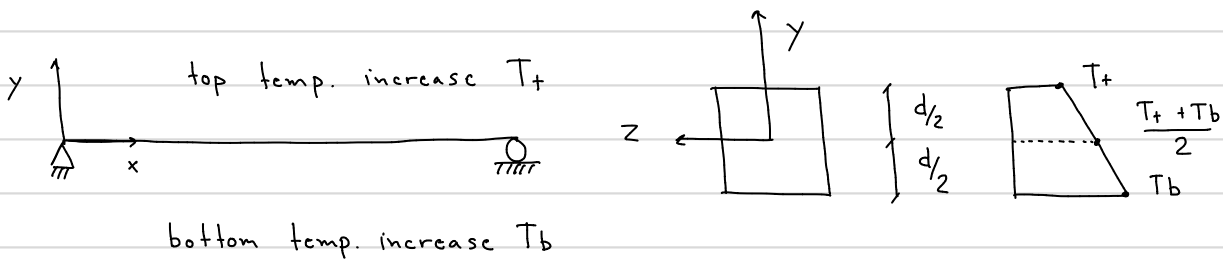
November 05, 2024
Matrix Displacement Method
Temperature Effects
(Plane Frames)

Temperature Changes

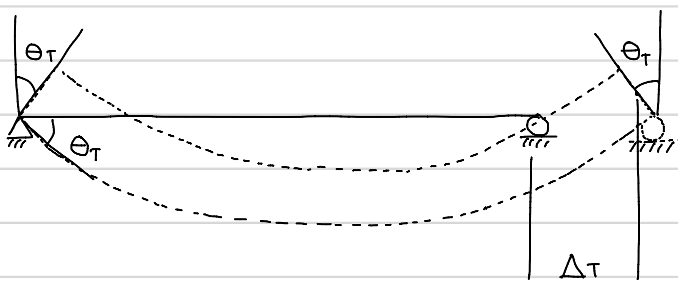
Structural effects from temperature changes can be incorporated into the MDM framework through modification of the local fixed-end force vector $\{Q_f\}$

We start by examining a plane frame element that is simply-supported (determinate) subjected to differential temperate increase :



$T_b > T_t$

* uniform along the length *



$T_b > T_t$

concave up

temperature increase results in overall elongation

temperature gradient through the depth results in bending

displacement = strain (ε) * original length (L)

$\alpha\,(T_{final} - T_{initial})$ $\quad \alpha \equiv$ coefficient of thermal expansion

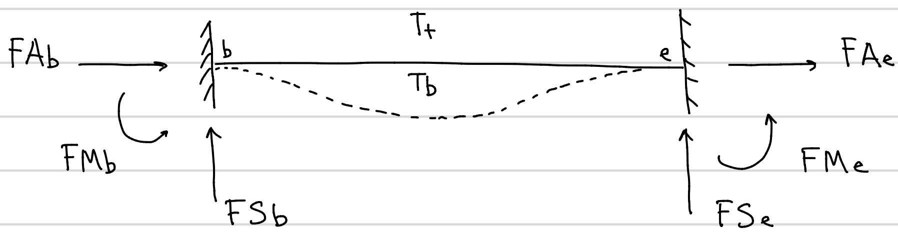
thus the elongation of the neutral axis $\Delta_T = \alpha \left(\dfrac{T_t + T_b}{2}\right) L$

rotations of the member ends $\theta_T = \dfrac{\alpha\,(T_b - T_t)\,L}{2d}$

$$\{u_T\} = \begin{Bmatrix} 0 \\ 0 \\ -\theta_T \\ \Delta_T \\ 0 \\ \theta_T \end{Bmatrix} = \frac{\alpha L}{2} \begin{Bmatrix} 0 \\ 0 \\ -(T_b - T_t)/d \\ (T_b + T_t) \\ 0 \\ (T_b - T_t)/d \end{Bmatrix}$$

We can determine the member fixed-end forces $\{Q_{fT}\}$ necessary to supress end displacements $\{u_T\}$ using principle of superposition

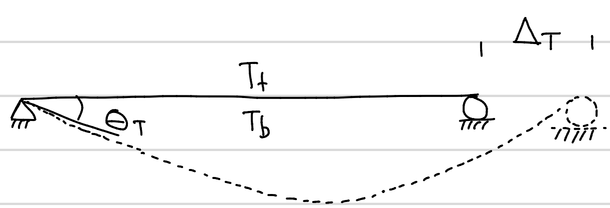
$u = 0$



temp. change and fixed-end forces

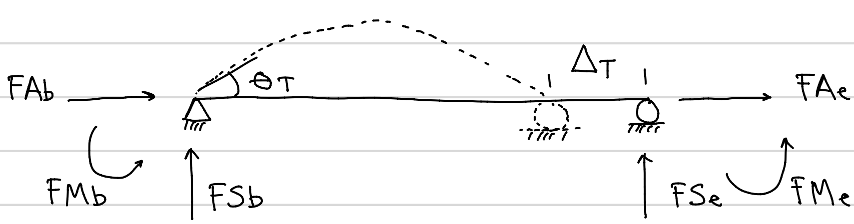
=

$u_I = u_T$



temp. change only

$u_{II} = -u_T$



fixed-end forces only

* Compatibility $u_I + u_{II} = 0 \quad \therefore \quad u_{II} = -u_I = -\{u_T\}$

Thus, $\{Q_{fT}\} = [k]\{-u_T\}$

$$\begin{Bmatrix} F_{Ab} \\ F_{Sb} \\ F_{Mb} \\ F_{Ae} \\ F_{Se} \\ F_{Me} \end{Bmatrix} = \frac{EI}{L^3}\begin{bmatrix} \frac{AL^2}{I} & 0 & 0 & -\frac{AL^2}{I} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ -\frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix} - \frac{\alpha L}{2}\begin{Bmatrix} 0 \\ 0 \\ -(T_b - T_t)/d \\ (T_b + T_t) \\ 0 \\ (T_b - T_t)/d \end{Bmatrix}$$

$$\{Q_{fT}\} = E\alpha\begin{Bmatrix} A(T_b + T_t)/2 \\ 0 \\ I(T_b - T_t)/d \\ -A(T_b + T_t)/2 \\ 0 \\ -I(T_b - T_t)/d \end{Bmatrix}$$

* For uniform temperature increase $T_b = T_t = T_u$

$F_{Ab} = -F_{Ae} = EA\alpha T_u$

$F_{Mb} = -F_{Me} = 0$ No bending

Above expressions can be used for beams, however, since axial forces are not considered $F_{Ab} = -F_{Ae} = 0$

Similarly, axial / truss structures can be modeled where $F_{Mb} = -F_{Me} = 0$

* Note: Truss equations must be modified to include $\{Q_f\}$

$\{Q\} = \{Q_f\} + [k]\{u\}$ where $\{Q_f\} = \{Q_{fT}\}$

$\{F\} = \{F_f\} + [K]\{v\}$ using transformations $[T]$

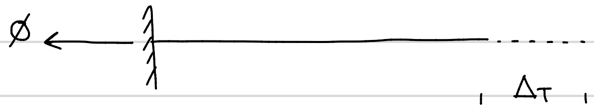
to solve $\{P - P_f\} = [S]\{d\}$ from assembly

** temperature effects in local $\{Q_{fT}\}$, support displacements in global $\{F_{fs}\} = \underbrace{[K]}_{\text{global}}\{v_s\}$

where $\{F_f\} = [T]^T\{\underbrace{Q_f}_{\text{interior loads}} + \underbrace{Q_{fT}}_{\text{temperature changes}}\} + \underbrace{\{F_{fs}\}}_{\text{support displacements}} + \underbrace{\ldots}_{\text{fabrication errors}}$, etc.

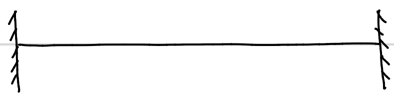
Lastly, how do boundary conditions affect structural response?

consider <u>uniform</u> temperature increase (T_u)



determinate

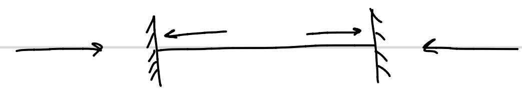
<u>axial</u> strain (αT_u), but no stresses



indeterminate

no strain, but stresses ($E \alpha T_u$)

<u>fixed-fixed</u> : temperature increase ↑ compression



decrease ↓ tension