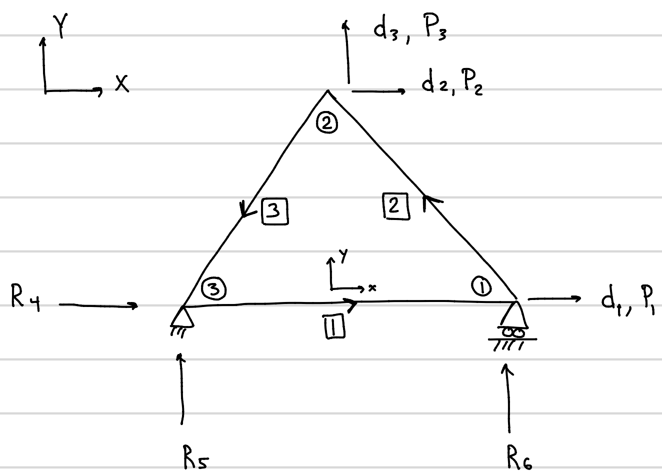
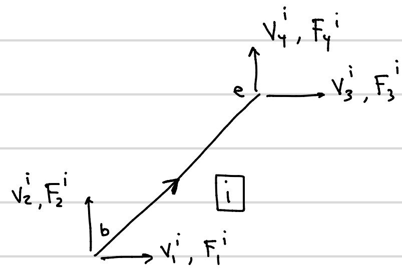
September 12, 2024
Matrix Displacement Method
2D Trusses
Part II - Assembly

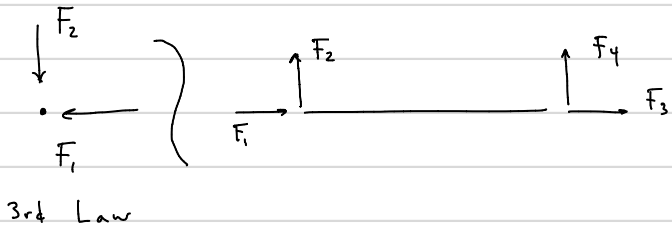
2D Truss Assembly

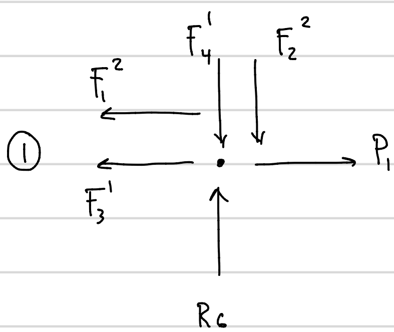




Rigorous Assembly

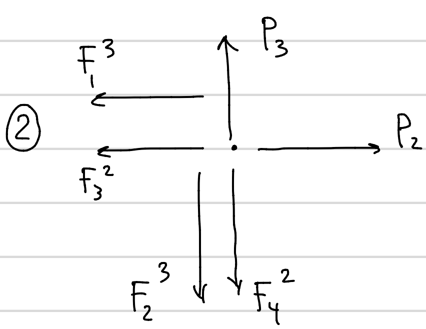
Equilibrium of Joints





$\sum F_x = 0 \qquad P_1 - F_3^1 - F_1^2 = 0 \qquad * P_1 = F_3^1 + F_1^2$

$\sum F_y = 0 \qquad R_6 - F_4^1 - F_2^2 = 0 \qquad R_6 = F_4^1 + F_2^2$



$\sum F_x = 0 \qquad P_2 - F_3^2 - F_1^3 = 0 \qquad * P_2 = F_3^2 + F_1^3$

$\sum F_y = 0 \qquad P_3 - F_4^2 - F_2^3 = 0 \qquad * P_3 = F_4^2 + F_2^3$

$$* \{P\} = [S]\{d\} *$$

$P_1 = F_3^1 + F_1^2 \qquad P_2 = F_3^2 + F_1^3 \qquad P_3 = F_4^2 + F_2^3$

$\{P\} = [S]\{d\}$ assemble from element $[K]$s

$\{F\} = [K]\{v\}$

$F_j^i = K_{j1}^i v_1^i + + K_{j2}^i v_2^i + K_{j3}^i v_3^i + K_{j4}^i v_4^i$

$$P_1 = \left(K_{31}^1 v_1^1 + K_{32}^1 v_2^1 + K_{33}^1 v_3^1 + K_{34}^1 v_4^1\right) + \left(K_{11}^2 v_1^2 + K_{12}^2 v_2^2 + K_{13}^2 v_3^2 + K_{14}^2 v_4^2\right)$$

$$P_2 = \left(K_{31}^2 v_1^2 + K_{32}^2 v_2^2 + K_{33}^2 v_3^2 + K_{34}^2 v_4^2\right) + \left(K_{11}^3 v_1^3 + K_{12}^3 v_2^3 + K_{13}^3 v_3^3 + K_{14}^3 v_4^3\right)$$

$$P_3 = \left(K_{41}^2 v_1^2 + K_{42}^2 v_2^2 + K_{43}^2 v_3^2 + K_{44}^2 v_4^2\right) + \left(K_{21}^3 v_1^3 + K_{22}^3 v_2^3 + K_{23}^3 v_3^3 + K_{24}^3 v_4^3\right)$$

$\{P\} = [S]\{d\} \longrightarrow$ compatibility w/ $\{v\}$

Joint ① $\quad d_1 = v_3^1 = v_1^2$

$v_4^1 = v_2^2 = 0 \quad$ support / boundary condition (B.C.)

Joint ② $\quad d_2 = v_3^2 = v_1^3$

$d_3 = v_4^2 = v_2^3$

Joint ③ $\quad v_1^1 = v_3^3 = 0 \quad$ B.C. X-dir.

$v_2^1 = v_4^3 = 0 \quad$ B.C. Y-dir.

$$P_1 = \overbrace{K_{33}^1 d_1 + K_{11}^2 d_1}^{S_{11}} + \overbrace{K_{13}^2 d_2}^{S_{12}} + \overbrace{K_{14}^2 d_3}^{S_{13}}$$

$$P_2 = K_{31}^2 d_1 + \overbrace{K_{33}^2 d_2 + K_{11}^3 d_2}^{S_{22}} + K_{34}^2 d_3 + K_{12}^3 d_3$$

$$P_3 = K_{41}^2 d_1 + K_{43}^2 d_2 + K_{21}^3 d_2 + \overbrace{K_{44}^2 d_3 + K_{22}^3 d_3}^{S_{33}}$$

$[K] = [T]^T [k] [T]$

symmetric $[M] = [M]^T$

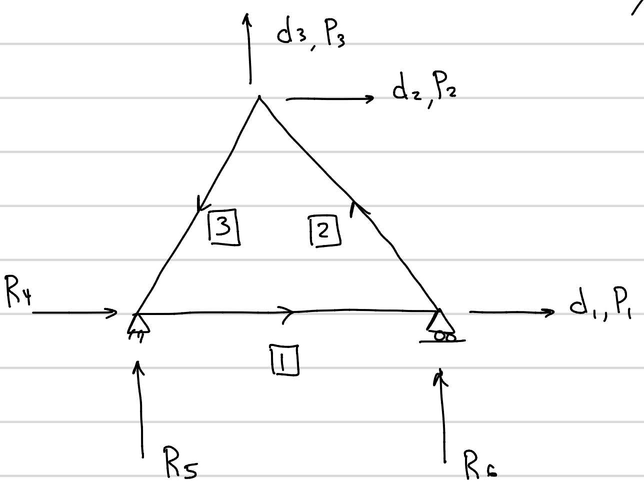
$[K]^T = \left([T]^T [k] [T]\right)^T$

$= [T]^T [k]^T \left([T]^T\right)^T \qquad [k]_{local}$ symmetric

$= [T]^T [k] [T]$

$\therefore [K]_{global}$ is symmetric

Code # Assembly



member # \ code #	F_1, V_1	F_2, V_2	F_3, V_3	F_4, V_4
1	4	5	1	6
2	1	6	2	3
3	2	3	4	5

$$[K]^1 = \begin{array}{c|cccc} & 4 & 5 & 1 & 6 \\ \hline 4 & K_{11} & K_{12} & K_{13} & K_{14} \\ 5 & K_{21} & K_{22} & K_{23} & K_{24} \\ 1 & K_{31} & K_{32} & K_{33} & K_{34} \\ 6 & K_{41} & K_{42} & K_{43} & K_{44} \end{array}$$

$$[S] = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & S_{11} & S_{12} & S_{13} \\ 2 & S_{21} & S_{22} & S_{23} \\ 3 & S_{31} & S_{32} & S_{33} \end{array}$$

$$[K]^2 = \begin{array}{c|cccc} & 1 & 6 & 2 & 3 \\ \hline 1 & K_{11} & K_{12} & K_{13} & K_{14} \\ 6 & K_{21} & K_{22} & K_{23} & K_{24} \\ 2 & K_{31} & K_{32} & K_{33} & K_{34} \\ 3 & K_{41} & K_{42} & K_{43} & K_{44} \end{array}$$

$$[S] = \begin{bmatrix} K_{33}^1 + K_{11}^2 & K_{13}^2 & K_{14}^2 \\ K_{31}^2 & K_{33}^2 + K_{11}^3 & K_{34}^2 + K_{12}^3 \\ K_{41}^2 & K_{43}^2 + K_{21}^3 & K_{44}^2 + K_{22}^3 \end{bmatrix}$$

$$[k]^3 = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & K_{11} & K_{12} & K_{13} & K_{14} \\ 3 & K_{21} & K_{22} & K_{23} & K_{24} \\ 4 & K_{31} & K_{32} & K_{33} & K_{34} \\ 5 & K_{41} & K_{42} & K_{43} & K_{44} \end{array}$$

<u>Solve</u> $\{d\} = [S]^{-1}\{P\}$

<u>Post-process</u> $\{F\} = [K]\{v\} \longrightarrow$ compatibility w/ $\{d\}$

$\{Q\} = [T]\{F\}$

<u>Alternatively</u>

$\{v\}$ from $\{d\}$ $\quad \{u\} = [T]\{v\}$ $\quad \{Q\} = [k]\{u\}$ $\quad \{F\} = [T]^T\{Q\}$ for reactions