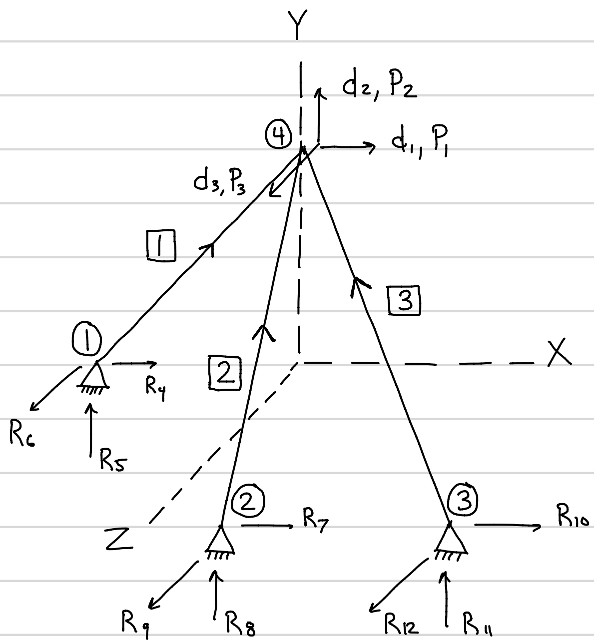
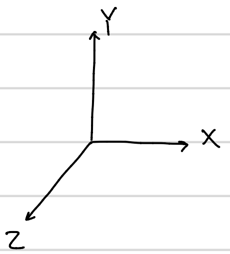
September 19, 2024
Matrix Displacement Method
3D Trusses
Part III - Formulation

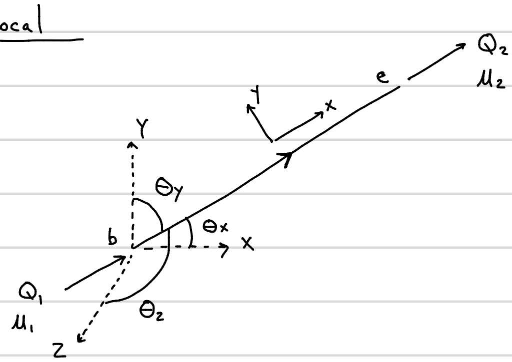
3D (Space) Trusses





global axes

Local



$$L = \sqrt{(X_e - X_b)^2 + (Y_e - Y_b)^2 + (Z_e - Z_b)^2}$$

$$\cos\theta_x = \frac{X_e - X_b}{L} \qquad \cos\theta_y = \frac{Y_e - Y_b}{L} \qquad \cos\theta_z = \frac{Z_e - Z_b}{L}$$

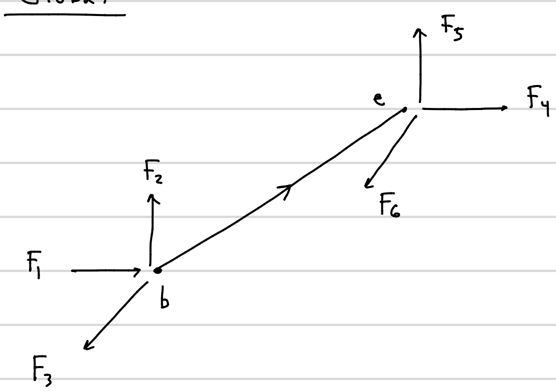
where X_b, X_e ; Y_b, Y_e ; Z_b, Z_e are coordinates

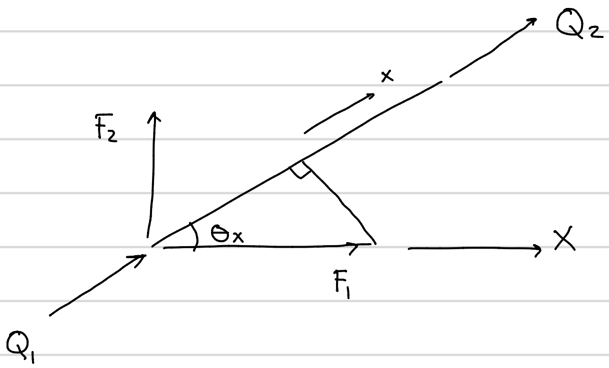
(sign (+/-) is important for direction cosines)

$$\{Q\} = [k]\{u\}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

Global





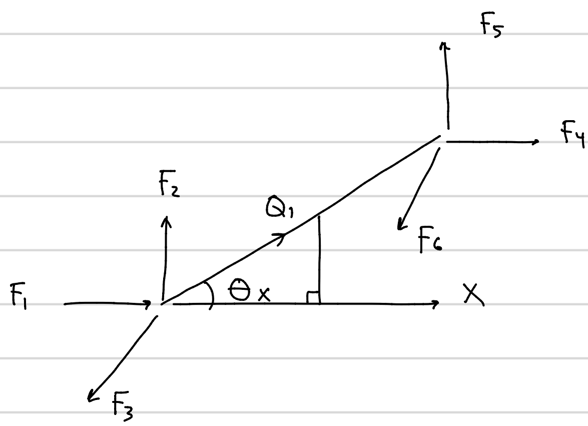
$$Q_1 = F_1 \cos\theta_x + F_2 \cos\theta_y + F_3 \cos\theta_z$$
$$Q_2 = F_4 \cos\theta_x + F_5 \cos\theta_y + F_6 \cos\theta_z$$

$$\underset{2\times 1}{\{Q\}} = \underset{2\times 6}{[T]} \underset{6\times 1}{\{F\}}$$

$$\begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \underbrace{\begin{bmatrix} \cos\theta_x & \cos\theta_Y & \cos\theta_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta_x & \cos\theta_Y & \cos\theta_z \end{bmatrix}}_{[T]} \begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix}$$

$$\{Q\} = [T]\{F\}$$

$$\{u\} = [T]\{v\}$$



$$F_1 = Q_1 \cos\theta x \qquad F_2 = Q_1 \cos\theta Y \qquad F_3 = Q_1 \cos\theta z$$

$$F_4 = Q_2 \cos\theta x \qquad F_5 = Q_2 \cos\theta Y \qquad F_6 = Q_2 \cos\theta z$$

$$\begin{Bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{Bmatrix} = \underbrace{\begin{bmatrix} \cos\theta x & 0 \\ \cos\theta Y & 0 \\ \cos\theta z & 0 \\ 0 & \cos\theta x \\ 0 & \cos\theta Y \\ 0 & \cos\theta z \end{bmatrix}}_{[T]^T} \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix}$$

$$\underset{6\times1}{\{F\}} = \underset{6\times2}{[T]^T} \underset{2\times1}{\{Q\}}$$

prove this !

$$\{F\} = \underbrace{[T]^T [k] [T]}_{[K]_{global}} \{v\}$$

$$\cancel{\{v\} = [T]^T \{u\}}$$

NOT DEFINED !

* u does not contain member end displacements in local y, z directions.
While member end forces Q are zero (negligible) in local y, z directions,
the local member end displacements are typically nonzero, but small compared to axial values.