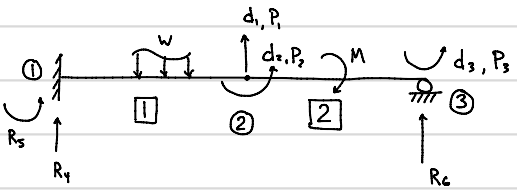
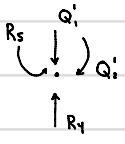
March 26, 2026
Matrix Displacement Method
Beams
Part III - Assembly

Assembly



Rigorous Assembly
(joint equilibrium)

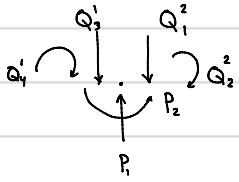
joint ①



$\Sigma F_y = 0 \quad R_A = Q_1'$

$\Sigma M_\cdot = 0 \quad R_S = Q_2'$

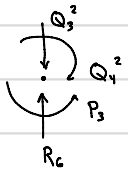
joint ②



$\Sigma F_y = 0 \quad P_1 = Q_3' + Q_1^2$ *

$\Sigma M_\cdot = 0 \quad P_2 = Q_4' + Q_2^2$ *

joint ③



$\Sigma F_y = 0 \quad R_C = Q_3^2$

$\Sigma M_\cdot = 0 \quad P_3 = Q_4^2$ *

$$\{Q\} = \{Q_f\} + [k]\{u\}$$

$P_1 = Q_3' + Q_1^2$

$P_2 = Q_4' + Q_2^2$

$P_3 = Q_4^2$

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \begin{Bmatrix} Qf_1 \\ Qf_2 \\ Qf_3 \\ Qf_4 \end{Bmatrix} + \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix}$$

$$P_1 = \left(Qf_3' + k_{31}'u_1' + k_{32}'u_2' + k_{33}'u_3' + k_{34}'u_4'\right) + \left(Qf_1^2 + k_{11}^2u_1^2 + k_{12}^2u_2^2 + k_{13}^2u_3^2 + k_{14}^2u_4^2\right)$$

$$P_2 = \left(Qf_4' + k_{41}'u_1' + k_{42}'u_2' + k_{43}'u_3' + k_{44}'u_4'\right) + \left(Qf_2^2 + k_{21}^2u_1^2 + k_{22}^2u_2^2 + k_{23}^2u_3^2 + k_{24}^2u_4^2\right)$$

$$P_3 = \left(Qf_4^2 + k_{41}^2u_1^2 + k_{42}^2u_2^2 + k_{43}^2u_3^2 + k_{44}^2u_4^2\right)$$

Compatibility

$u_1^1 = u_2^1 = 0$ fixed support (B.C.)

$u_3^1 = u_1^2 = d_1$

$u_4^1 = u_2^2 = d_2$

$u_3^2 = 0$ roller support (B.C.)

$u_4^2 = d_3$

Substitute compatibility relations into joint equilibrium equations

$$P_1 = Qf_3^1 + k_{33}^1 d_1 + k_{34}^1 d_2 + Qf_1^2 + k_{11}^2 d_1 + k_{12}^2 d_2 + k_{14}^2 d_3$$

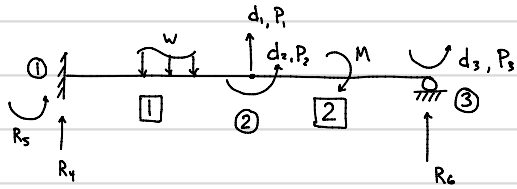
$$P_2 = Qf_4^1 + k_{43}^1 d_1 + k_{44}^1 d_2 + Qf_2^2 + k_{21}^2 d_1 + k_{22}^2 d_2 + k_{24}^2 d_3$$

$$P_3 = Qf_4^2 + k_{41}^2 d_1 + k_{42}^2 d_2 + k_{44}^2 d_3$$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \underbrace{\begin{Bmatrix} Qf_3^1 + Qf_1^2 \\ Qf_4^1 + Qf_2^2 \\ Qf_4^2 \end{Bmatrix}}_{\{P_f\}} + \underbrace{\begin{bmatrix} k_{33}^1 + k_{11}^2 & k_{34}^1 + k_{12}^2 & k_{14}^2 \\ k_{43}^1 + k_{21}^2 & k_{44}^1 + k_{22}^2 & k_{24}^2 \\ k_{41}^2 & k_{42}^2 & k_{44}^2 \end{bmatrix}}_{[S]} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

$$\{P - P_f\} = [S]\{d\}$$

Code # Assembly



member # \ code #	M_1, Q_1	M_2, Q_2	M_3, Q_3	M_4, Q_4
1	4	5	1	2
2	1	2	6	3

1

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} \begin{matrix} 4 \\ 5 \\ 1 \\ 2 \end{matrix} = \begin{Bmatrix} Qf_1 \\ Qf_2 \\ Qf_3 \\ Qf_4 \end{Bmatrix} + \begin{matrix} 4 \\ 5 \\ 1 \\ 2 \end{matrix} \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{Bmatrix}$$

(4 5 1 2)

2

$$\begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 6 \\ 3 \end{matrix} = \begin{Bmatrix} Qf_1 \\ Qf_2 \\ Qf_3 \\ Qf_4 \end{Bmatrix} + \begin{matrix} 1 \\ 2 \\ 6 \\ 3 \end{matrix} \begin{bmatrix} k_{11} & k_{12} & k_{13} & k_{14} \\ k_{21} & k_{22} & k_{23} & k_{24} \\ k_{31} & k_{32} & k_{33} & k_{34} \\ k_{41} & k_{42} & k_{43} & k_{44} \end{bmatrix} \begin{Bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \end{Bmatrix}$$

(1 2 6 3)

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} = \begin{Bmatrix} Qf_3^{1} + Qf_1^{2} \\ Qf_4^{1} + Qf_2^{2} \\ Qf_4^{2} \end{Bmatrix} + \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \begin{bmatrix} k_{33}^{1} + k_{11}^{2} & k_{34}^{1} + k_{12}^{2} & k_{14}^{2} \\ k_{43}^{1} + k_{21}^{2} & k_{44}^{1} + k_{22}^{2} & k_{24}^{2} \\ k_{41}^{2} & k_{42}^{2} & k_{44}^{2} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

(1 2 3)

solution $\{d\} = [S]^{-1}\{P - P_f\}$

post-process $\{Q\} = \{Qf\} + [k]\{M\}$ ↓ compatibility w/ $\{d\}$

Compute reactions $\{R\}$ from joint equilibrium

Shear and Moment diagrams ⟶ compute stresses Normal/bending $\sigma_b = \frac{-My}{I}$ Shear $\tau = \frac{VQ}{Ib}$