

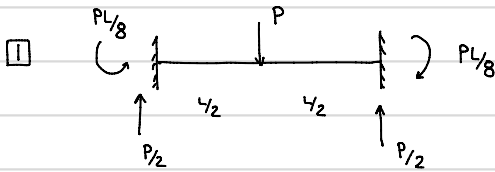
$E = 4,000 \text{ ksi}$ ,  $I = 1500 \text{ in}^4$  for all members

$EI = 6e6 \text{ k}\cdot\text{in}^2$

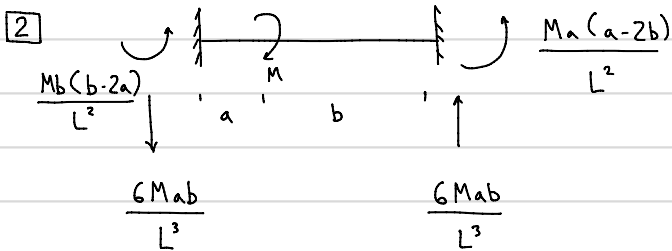
2. Create  $\{P\}$

$$\{P\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \text{NO loads applied at DOFs}$$

3. Determine  $\{Q_f\}$ ,  $[k]$



$$\{Q_f\}^1 = \begin{Bmatrix} P/2 \\ PL/8 \\ P/2 \\ -PL/8 \end{Bmatrix} = \begin{Bmatrix} 18 \\ 864 \\ 18 \\ -864 \end{Bmatrix} \begin{matrix} \text{k} \\ \text{k}\cdot\text{in} \\ \text{k} \\ \text{k}\cdot\text{in} \end{matrix}$$



$$\{Q_f\}^2 = \begin{Bmatrix} -\frac{6Mab}{L^3} \\ \frac{Mb(b-2a)}{L^2} \\ \frac{6Mab}{L^3} \\ \frac{Ma(a-2b)}{L^2} \end{Bmatrix} = \begin{Bmatrix} -1.125 \\ 18 \\ 1.125 \\ -30 \end{Bmatrix} \begin{matrix} \text{k} \\ \text{k}\cdot\text{in} \\ \text{k} \\ \text{k}\cdot\text{in} \end{matrix}$$

$$[k] = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \begin{matrix} u_1 & \Delta & \text{in.} \\ u_2 & \Theta & \text{rad} \\ u_3 & \Delta & \text{in.} \\ u_4 & \Theta & \text{rad.} \end{matrix}$$

$$\{Q\} = \{Q_f\} + [k]\{u\}$$

$$\square \quad L = 16 \times 12 = 192 \text{ in.} \quad \frac{12EI}{L^3} = 10.173 \text{ k/in.} \quad \frac{4EI}{L} = 1.25e5 \text{ k}\cdot\text{in}$$

$$EI = 6e6 \text{ k}\cdot\text{in}^2$$

$$\frac{6EI}{L^2} = 976.56 \text{ k}$$

$$\frac{2EI}{L} = 0.625e5 \text{ k}\cdot\text{in}$$

$$\boxed{2} \quad L = 8 \times 12 = 96''$$

$$EI = 6e6 \text{ k}\cdot\text{in}^2$$

$$\frac{12EI}{L^3} = 81.38 \text{ k/in} \quad \frac{4EI}{L} = 2.5e5 \text{ k}\cdot\text{in}$$

$$\frac{6EI}{L^2} = 3906.2 \text{ k} \quad \frac{2EI}{L} = 1.25e5 \text{ k}\cdot\text{in}$$

#### 4. Assembly (Code #)

code # member #	$M_1, Q_1$	$M_2, Q_2$	$M_3, Q_3$	$M_4, Q_4$
1	4	5	1	2
2	1	2	6	3

$$\{P_f\} = \begin{Bmatrix} P_{f1} \\ P_{f2} \\ P_{f3} \end{Bmatrix} = \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \begin{Bmatrix} Q_{f3}' + Q_{f1}'' \\ Q_{f4}' + Q_{f2}'' \\ Q_{f1}'' \end{Bmatrix}$$

$$[S] = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} k_{33}' + k_{11}'' & k_{34}' + k_{12}'' & k_{14}'' \\ k_{43}' + k_{21}'' & k_{44}' + k_{22}'' & k_{24}'' \\ k_{41}'' & k_{42}'' & k_{44}'' \end{bmatrix} \end{matrix}$$

$$\{P_f\} = \begin{Bmatrix} 16.875 \\ -846 \\ -30 \end{Bmatrix} \begin{matrix} \text{k} \\ \text{k}\cdot\text{in} \\ \text{k}\cdot\text{in} \end{matrix}$$

$$[S] = \begin{bmatrix} 91.56 & 2929.7 & 3906.2 \\ 2929.7 & 3.75e5 & 1.25e5 \\ 3906.2 & 1.25e5 & 2.5e5 \end{bmatrix}$$

#### 5. Solve for $\{d\}$

$$\{P - P_f\} = [S]\{d\} \quad \therefore \{d\} = [S]^{-1}\{P - P_f\} \quad \{d\} = \begin{Bmatrix} -0.726 \\ 0.00493 \\ 0.009 \end{Bmatrix} \begin{matrix} \text{in.} \\ \text{rad} \\ \text{rad} \end{matrix}$$

#### 6. Compute $\{Q\}$

$$\{Q\} = \underbrace{\{Q_f\}}_{\text{known}} + \underbrace{[k]}_{\text{known}} \underbrace{\{u\}}_{\text{known}} \quad \text{compatibility}$$

$$\{u\}' = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ d_1 \\ d_2 \end{Bmatrix} \begin{matrix} \text{fixed B.C.} \\ \text{fixed B.C.} \\ -0.726 \\ 0.00493 \end{matrix}$$

$$\{u\}'' = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{Bmatrix} = \begin{Bmatrix} d_1 \\ d_2 \\ 0 \\ d_3 \end{Bmatrix} \begin{matrix} -0.726 \\ 0.00493 \\ \text{roller B.C.} \\ 0.009 \end{matrix}$$

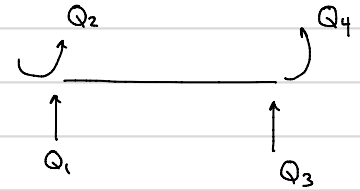
$$\{Q\}^1 = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \begin{Bmatrix} 30.198 \\ 1881 \\ 5.8021 \\ 461 \end{Bmatrix} \begin{matrix} k \\ k \cdot \text{in} \\ k \\ k \cdot \text{in} \end{matrix}$$

$$\{Q\}^2 = \begin{Bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \end{Bmatrix} = \begin{Bmatrix} -5.8021 \\ -461 \\ 5.8021 \\ -2.34 \cdot 10^{-13} \end{Bmatrix} \begin{matrix} k \\ k \cdot \text{in} \\ k \\ k \cdot \text{in} \end{matrix} \approx 0$$

### 7. Reactions

joint ①

$\sum F_y = 0 \quad R_y = Q_1 = 30.198 \text{ k}$   
 $\sum M = 0 \quad R_s = Q_2 = 156.75 \text{ k} \cdot \text{ft}$

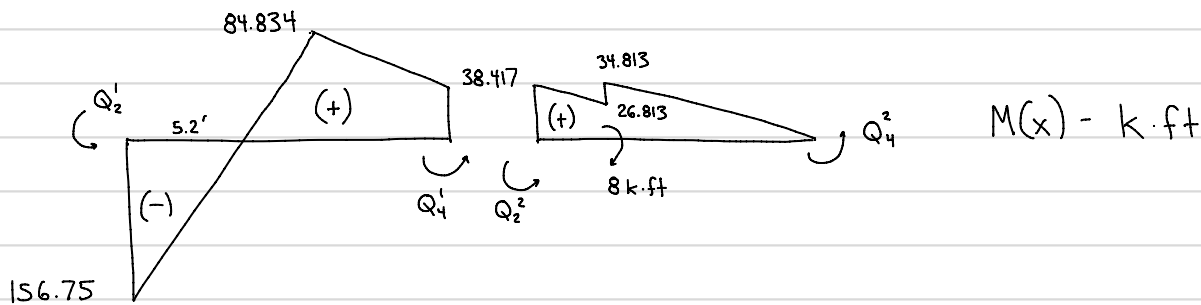
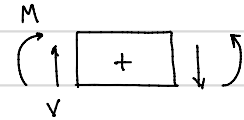
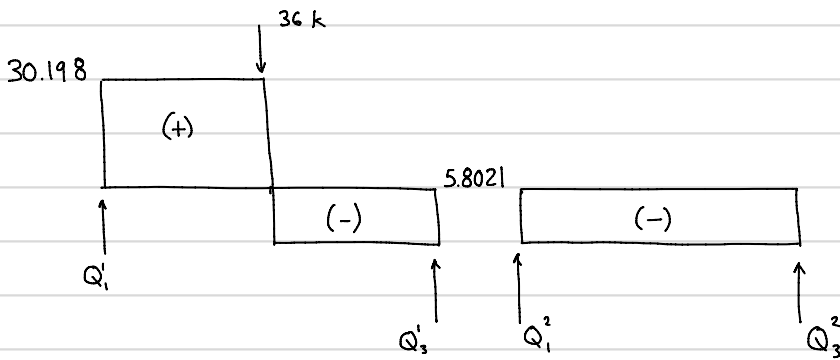


joint ③

$\sum F_y = 0 \quad R_c = Q_3 = 5.8021 \text{ k}$   
 $\sum M = 0 \quad Q_4 = 0 \quad (-2.34 \cdot 10^{-13}) \text{ numerical error}$

\* Quick check : Verify overall equilibrium \*

### 8. Draw shear force and bending moment diagrams



9. Calculate Stresses - Normal/bending:  $\sigma_b = \frac{-M_y}{I}$  Shear:  $\tau = \frac{VQ}{Ib}$