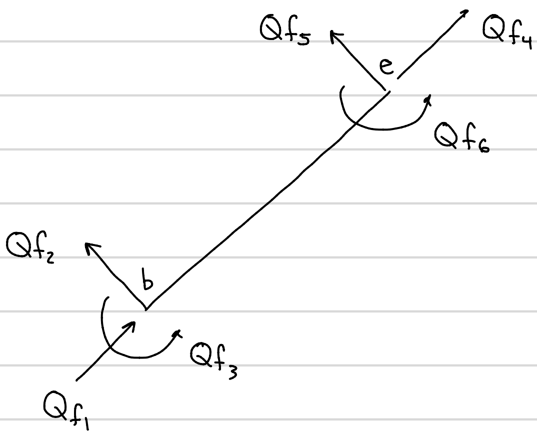
April 09, 2026
Matrix Displacement Method
2D Frames
Part II - Formulation cont'd

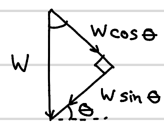
Fixed-end forces (local coordinates)

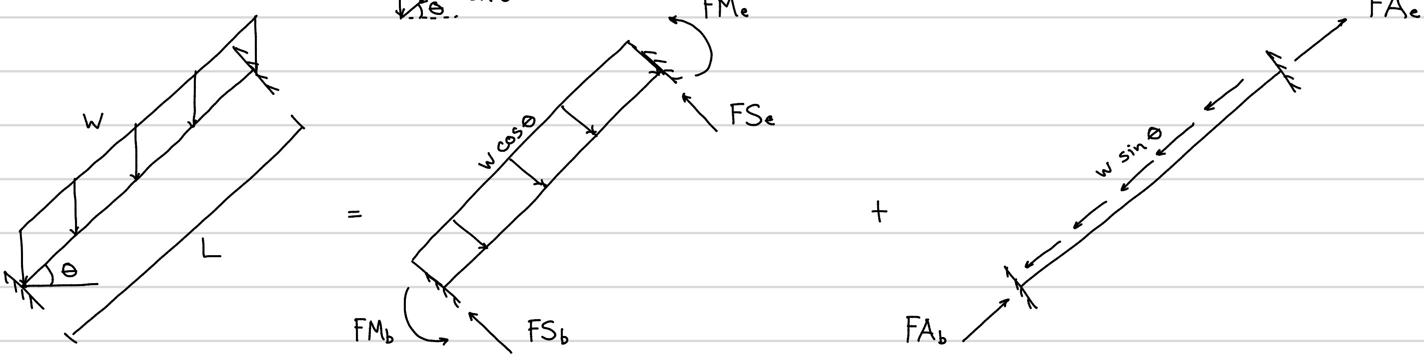


$$\{Q_f\} = \begin{array}{c} \begin{Bmatrix} FA_b \\ FS_b \\ FM_b \\ FA_e \\ FS_e \\ FM_e \end{Bmatrix} \end{array} \begin{array}{l} \text{axial} \\ \text{shear} \\ \text{moment} \\ \\ \\ \\ \end{array}$$

Kassimali Notation

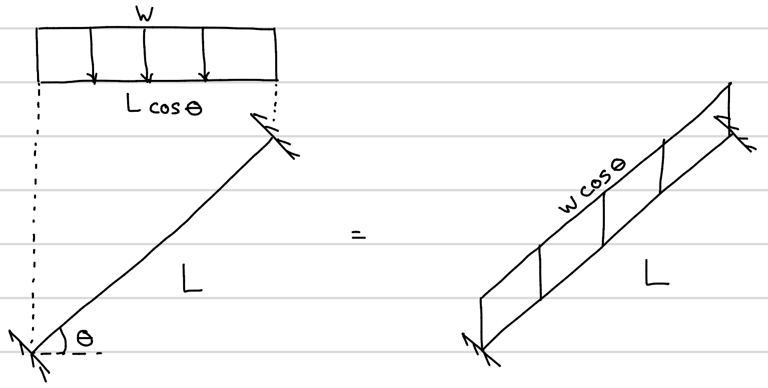
Self-weight loads





$$\{Q_f\} = \begin{Bmatrix} \frac{1}{2}\, wL \sin\theta \\ \frac{1}{2}\, wL \cos\theta \\ \frac{1}{12}\, wL^2 \cos\theta \\ \frac{1}{2}\, wL \sin\theta \\ \frac{1}{2}\, wL \cos\theta \\ -\frac{1}{12}\, wL^2 \cos\theta \end{Bmatrix} = \frac{wL}{2} \begin{Bmatrix} \sin\theta \\ \cos\theta \\ \frac{1}{6} L \cos\theta \\ \sin\theta \\ \cos\theta \\ -\frac{1}{6} L \cos\theta \end{Bmatrix}$$

Projected loads (e.g. snow loading)

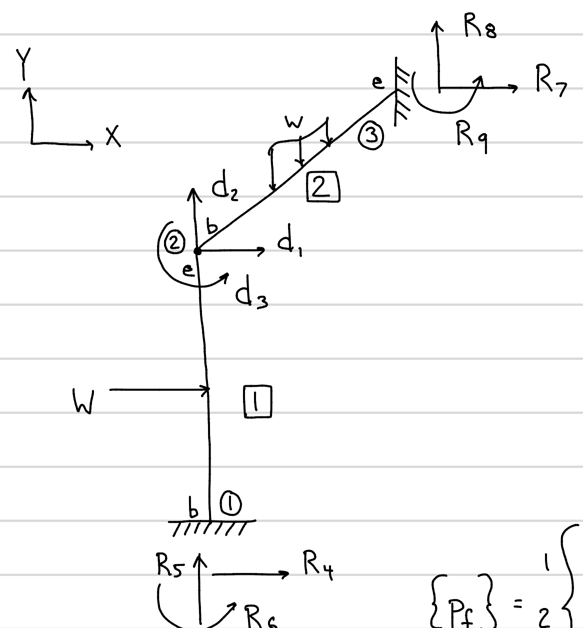


$$\{Q_f\} = \frac{wL\cos\theta}{2} \begin{Bmatrix} \sin\theta \\ \cos\theta \\ \frac{1}{6} L\cos\theta \\ \sin\theta \\ \cos\theta \\ -\frac{1}{6} L\cos\theta \end{Bmatrix}$$

$$\sin\theta = L_y / L$$
$$\cos\theta = L_x / L$$

Once $\underset{\text{local}}{\{Q_f\}}$ is obtained, $\underset{\text{global}}{\{F_f\}} = [T]^T \{Q_f\}$

Now assembly (via code # method)



member # \ code #	V_1, F_1	V_2, F_2	V_3, F_3	V_4, F_4	V_5, F_5	V_6, F_6
1	4	5	6	1	2	3
2	1	2	3	7	8	9

$$\{P\} = \{P_f\} + [S]\{d\}$$

$$\{F_f\}^1 = \begin{array}{c} 4 \\ 5 \\ 6 \\ 1 \\ 2 \\ 3 \end{array} \begin{Bmatrix} F_{f_1}^1 \\ F_{f_2}^1 \\ F_{f_3}^1 \\ F_{f_4}^1 \\ F_{f_5}^1 \\ F_{f_6}^1 \end{Bmatrix} \qquad \{F_f\}^2 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 7 \\ 8 \\ 9 \end{array} \begin{Bmatrix} F_{f_1}^2 \\ F_{f_2}^2 \\ F_{f_3}^2 \\ F_{f_4}^2 \\ F_{f_5}^2 \\ F_{f_6}^2 \end{Bmatrix}$$

$$\underline{\{P_f\}} = \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{Bmatrix} F_{f_4}^1 + F_{f_1}^2 \\ F_{f_5}^1 + F_{f_2}^2 \\ F_{f_6}^1 + F_{f_3}^2 \end{Bmatrix}$$

$$\underset{\text{global}}{[K]} = [T]^T \underset{\text{local}}{[k]} [T]$$

$$[K]^1 = \begin{array}{c} \\ 4 \\ 5 \\ 6 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} 4 & 5 & 6 & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \end{array}$$

$$[K]^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 7 \\ 8 \\ 9 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 7 & 8 & 9 \end{array} \\ \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \end{array}$$

$$[S] = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} K^1_{44} + K^2_{11} & K^1_{45} + K^2_{12} & K^1_{46} + K^2_{13} \\ K^1_{54} + K^2_{21} & K^1_{55} + K^2_{22} & K^1_{56} + K^2_{23} \\ K^1_{64} + K^2_{31} & K^1_{65} + K^2_{32} & K^1_{66} + K^2_{33} \end{bmatrix} \end{array}$$

Now that we have $\{P_f\}$ and $[S]$ we can solve * $\{P - P_f\} = [S]\{d\}$ *

$$\begin{array}{lcc} & \uparrow & \uparrow \\ \text{Assembly} & \{F_f\} & [K] \\ & \uparrow & \uparrow \\ \text{Transformation} & [T]^T\{Q_f\} & [T]^T[k][T] \end{array}$$