Matrix Displacement Method (MDM) Summary

Uniaxial: One (1) *structural-level* Degree of freedom (DOF): d_x – axial displacement

$$\{P\}=\{P_f\}+[S]\{d\}$$

Member end forces {Q}: 2 (axial forces 1,2) $\{Q\}=\{Q_f\}+[k]\{u\}$

Member stiffness matrix: [k] 2 x 2 $[k]=\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

Solution Steps

1. Label all members, joints, DOFs (d_x), and support reactions.
2. Create the *structural* force vector {P}
3. Determine all *member* stiffness matrices [k], and fixed end force vectors $\{Q_f\}$
4. Use the Code Number method to assemble the *structural* stiffness matrix [S] and fixed end force vector $\{P_f\}$
5. Solve for the DOFs {d} using $\{d\} = [S]^{-1}\{P-P_f\}$
6. Computer *member end* forces using $\{Q\} = \{Q_f\} + [k]\{u\}$ and compatibility of {d}-{u}
7. Determine support **reactions** from joint equilibrium
8. Draw *axial force* diagrams, determine axial **stress** from $\sigma=\frac{P}{A}$

2D Truss: Two (2) *structural-level* DOFs: d_X, d_Y – X,Y displacements

$$\{P\}=[S]\{d\}$$

<u>Member end forces:</u>

Local - {Q}: 4 (axial 1,3 & transverse forces 2,4) $\{Q\}=[k]\{u\}$

Global - {F}: 4 (X 1,3 & Y 2,4 forces) $\{F\}=[K]\{v\}$

{Q} = [T]{F}, {u} = [T]{v}

$$[T]=\begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}$$

<u>Member stiffness matrix:</u>

Local - [k] 4 x 4

$$[k]=\frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Global - $[K]=[T]^T[k][T]$

$$[K]=\frac{EA}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

Solution Steps

1. Label all members (w/ local coordinate axes), joints, DOFs (d_x,d_y), and support reactions.
2. Create the *structural* force vector {P}
3. Determine all *member* stiffness matrices in ***local*** coordinates [k], transformation matrices [T], and ***global*** stiffness matrices via $[K] = [T]^T[k][T]$.
4. Use the Code Number method to assemble the *structural* stiffness matrix [S]
5. Solve for the DOFs {d} using $\{d\} = [S]^{-1}\{P\}$
6. Computer *member end* forces in ***global*** coordinates using {F} = [K]{v} and compatibility of {d}-{v}
7. Calculate axial "**bar**" forces in each member:
   Option 1: {Q} = [T]{F} and report Q_3 since (+) is tension and (-) is compression
   Option 2: Use Pythagorean Theorem with F_1, F_2 to determine local axial force
8. Determine support **reactions** from joint equilibrium
9. Determine axial **stress** from $\sigma=\frac{P}{A}$

Beam: Two (2) *structural-level* DOFs: d_v, d_θ – transverse displacement, rotation

$$\{P\}=\{P_f\}+[S]\{d\}$$

Member end forces {Q}: 4 (transverse forces 1,3 – rotations 2,4)

$$\{Q\}=\{Q_f\}+[k]\{u\}$$

Member stiffness matrix: [k] 4 x 4

$$[k]=\frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Solution Steps

1. Label all members, joints, DOFs (d_v,d_θ), and support reactions.
2. Create the *structural* force vector {P}
3. Determine all *member* stiffness matrices [k], and fixed end force vectors $\{Q_f\}$
4. Use the Code Number method to assemble the *structural* stiffness matrix [S] and fixed end force vector $\{P_f\}$
5. Solve for the DOFs {d} using $\{d\} = [S]^{-1}\{P-P_f\}$
6. Computer *member end* forces using $\{Q\} = \{Q_f\} + [k]\{u\}$ and compatibility of {d}-{u}
7. Determine support **reactions** from joint equilibrium
8. Draw *shear force* and *bending moment* diagrams
9. Determine shear and bending **stresses** from $\tau=\frac{VQ}{Ib}$, $\sigma=-\frac{My}{I}$

2D Frame: Three (3) *structural-level* DOFs: d_X, d_Y, d_θ – XY displacements, rotation

$$\{P\}=\{P_f\}+[S]\{d\}$$

Member end forces:

Local - {Q}: 6 (axial forces 1,4 - transverse forces 2,5 – rotations 3,6) $\{Q\}=\{Q_f\}+[k]\{u\}$

Global - {F}: 6 (X forces 1,4 - Y forces 2,5 – rotations 3,6) $\{F\}=\{F_f\}+[K]\{v\}$

{Q} = [T]{F}, {u} = [T]{v}

$$[T]=\begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Member stiffness matrix:

Local – [k] 6 x 6

$$[k]=\begin{bmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix}$$

Solution Steps

1. Label all members, joints, DOFs (d_x,d_y,d_θ), and support reactions.
2. Create the *structural* force vector {P}
3. Determine all *member* stiffness matrices [k] and fixed end force vectors $\{Q_f\}$ in ***local*** coordinates, transformation matrices [T], and ***global*** stiffness matrices and global force vectors via $[K] = [T]^T[k][T]$, $\{F_f\} = [T]^T\{Q_f\}$.
4. Use the Code Number method to assemble the *structural* stiffness matrix [S] and fixed end force vector $\{P_f\}$
5. Solve for the DOFs {d} using $\{d\} = [S]^{-1}\{P-P_f\}$
6. Computer *member end* forces in ***global*** coordinates using $\{F\} = \{F_f\} + [K]\{v\}$ and compatibility of {d}-{v}
7. Determine support **reactions** from joint equilibrium
8. Draw *axial*, *shear force* and *bending moment* diagrams
9. Determine axial, shear and bending **stresses** from $\sigma_a = \frac{P}{A}$ $\tau = \frac{VQ}{Ib}$, $\sigma_b = -\frac{My}{I}$