Matrix Displacement Method (MDM) Summary

Uniaxial: One (1) *structural-level* Degree of freedom (DOF): d_x – axial displacement

$$\{P\}=\{P_f\}+[S]\{d\}$$

Member end forces {Q}: 2 (axial forces 1,2) $$\{Q\}=\{Q_f\}+[k]\{u\}$$

Member stiffness matrix: [k] 2 x 2 $$[k]=\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Solution Steps

1. Label all members, joints, DOFs (d_x), and support reactions.
2. Create the *structural* force vector {P}
3. Determine all *member* stiffness matrices [k], and fixed end force vectors $\{Q_f\}$
4. Use the Code Number method to assemble the *structural* stiffness matrix [S] and fixed end force vector $\{P_f\}$
5. Solve for the DOFs {d} using $\{d\} = [S]^{-1}\{P-P_f\}$
6. Computer *member end* forces using $\{Q\} = \{Q_f\} + [k]\{u\}$ and compatibility of {d}-{u}
7. Determine support **reactions** from joint equilibrium
8. Draw *axial force* diagrams, determine axial **stress** from $\sigma=\dfrac{N}{A}$

2D Truss: Two (2) *structural-level* DOFs: d_X, d_Y – X,Y displacements

$$\{P\}=[S]\{d\}$$

Member end forces:

Local - {Q}: 4 (axial 1,3 & transverse forces 2,4) $\{Q\}=[k]\{u\}$

Global - {F}: 4 (X 1,3 & Y 2,4 forces) $\{F\}=[K]\{v\}$

{Q} = [T]{F}, {u} = [T]{v}

$$[T]=\begin{bmatrix} c & s & 0 & 0 \\ -s & c & 0 & 0 \\ 0 & 0 & c & s \\ 0 & 0 & -s & c \end{bmatrix}$$

Member stiffness matrix:

Local - [k] 4 x 4

$$[k]=\frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Global - $[K]=[T]^T[k][T]$

$$[K]=\frac{EA}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

Solution Steps

1. Label all members (w/ local coordinate axes), joints, DOFs (d_X,d_Y), and support reactions.
2. Create the *structural* force vector {P}
3. Determine all *member* stiffness matrices in ***local*** coordinates [k], transformation matrices [T], and ***global*** stiffness matrices via $[K] = [T]^T[k][T]$.
4. Use the Code Number method to assemble the *structural* stiffness matrix [S]
5. Solve for the DOFs {d} using $\{d\} = [S]^{-1}\{P\}$
6. Computer *member end* forces in ***global*** coordinates using {F} = [K]{v} and compatibility of {d}-{v}
7. Calculate axial "**bar**" forces in each member:
   Option 1: {Q} = [T]{F} and report Q_3 since (+) is tension and (-) is compression
   Option 2: Use Pythagorean Theorem with F_1, F_2 to determine local axial force
8. Determine support **reactions** from joint equilibrium
9. Determine axial **stress** from $\sigma=\dfrac{N}{A}$

3D Spatial Truss: Three (3) *structural-level* DOFs: d_X, d_Y, d_Z – X,Y,Z displacements

$$\{P\}=[S]\{d\}$$

<u>Member end forces:</u>

Local - {Q}: 2 (axial forces 1,2) $\{Q\}=[k]\{u\}$

Global - {F}: 6 (X 1,4 & Y 2,5 & Z 3,6 forces) $\{F\}=[K]\{v\}$

{Q} = [T]{F}, {u} = [T]{v}

$$[T]=\begin{bmatrix} \cos\theta_X & \cos\theta_Y & \cos\theta_Z & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta_X & \cos\theta_Y & \cos\theta_Z \end{bmatrix}$$

<u>Member stiffness matrix:</u>

Local - [k] 2 x 2

$$[k]=\frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Global - $[K]=[T]^T[k][T]$

$$[K]=\frac{EA}{L}\begin{bmatrix} c^2\theta_X & c\theta_X c\theta_Y & c\theta_X c\theta_Z & -c^2\theta_X & -c\theta_X c\theta_Y & -c\theta_X c\theta_Z \\ - & c^2\theta_Y & c\theta_Y c\theta_Z & -c\theta_X c\theta_Y & -c^2\theta_Y & -c\theta_Y c\theta_Z \\ - & - & c^2\theta_Z & -c\theta_X c\theta_Z & -c\theta_Y c\theta_Z & -c^2\theta_Z \\ - & - & - & c^2\theta_X & c\theta_X c\theta_Y & c\theta_X c\theta_Z \\ - & - & - & - & c^2\theta_Y & c\theta_Y c\theta_Z \\ - & - & - & - & - & c^2\theta_Z \end{bmatrix}$$

<u>Solution Steps</u>

1. Label all members (w/ local coordinate axes), joints, DOFs (d_X,d_Y,d_Z), and support reactions.
2. Create the *structural* force vector {P}
3. Determine all *member* stiffness matrices in ***local*** coordinates [k], transformation matrices [T], and ***global*** stiffness matrices via $[K] = [T]^T[k][T]$.
4. Use the Code Number method to assemble the *structural* stiffness matrix [S]
5. Solve for the DOFs {d} using $\{d\} = [S]^{-1}\{P\}$
6. Computer *member end* forces in ***global*** coordinates using {F} = [K]{v} and compatibility of {d}-{v}
7. Calculate axial "**bar**" forces in each member:

   Option 1: {Q} = [T]{F} and report Q_2 since (+) is tension and (-) is compression
   Option 2: Use Pythagorean Theorem with F_1, F_2, F_3 to determine local axial force
8. Determine support **reactions** from joint equilibrium
9. Determine axial **stress** from $\sigma=\dfrac{N}{A}$