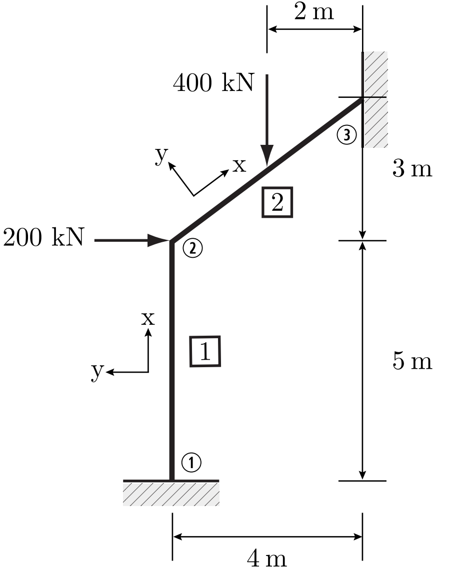
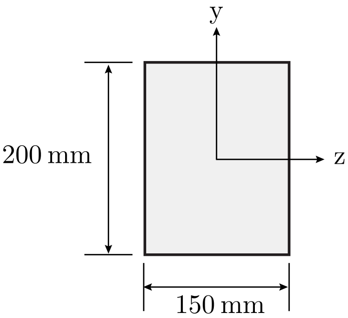


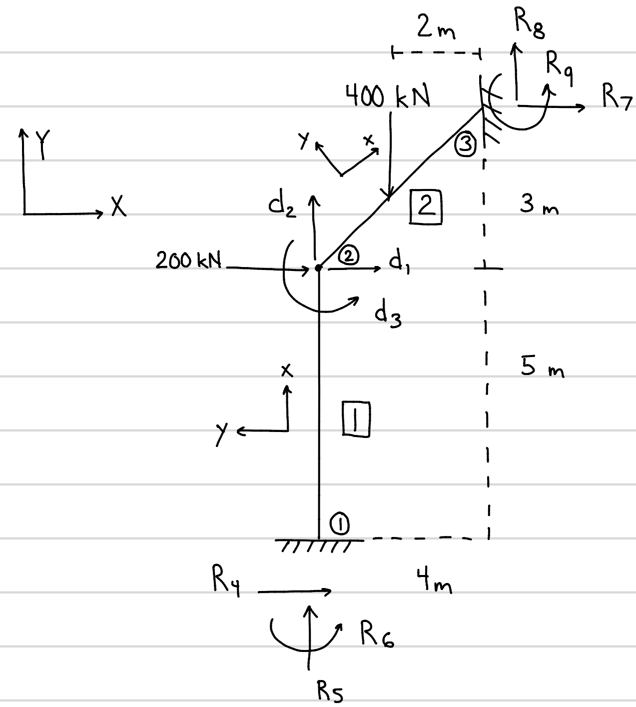


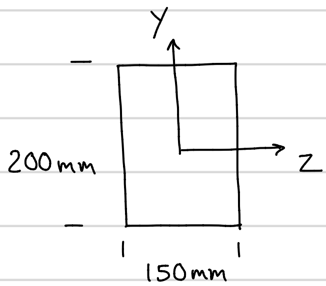
E = 20 GPa for all members

Plane Frame Example

2D Frame Example

1. Label Structure





(strong axis bending)

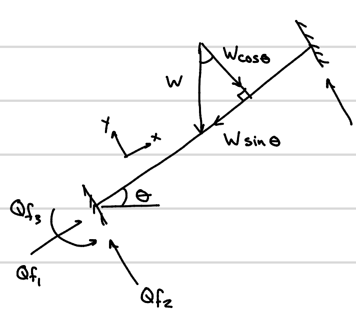
E = 20 GPa for all members

2.

$$\{P\}_{3\times1} = \begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} 200 \\ 0 \\ 0 \end{Bmatrix} \text{ kN}$$

3. Member-level contributions (local)

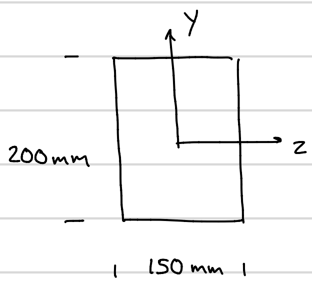
$$\{Q_f\}^1 = \{\emptyset\}$$



$$\{Q_f\}^2 = \begin{Bmatrix} FA_b \\ FS_b \\ FM_b \\ FA_e \\ FS_e \\ FM_e \end{Bmatrix} = \begin{Bmatrix} \frac{1}{2} W \sin\theta \\ \frac{1}{2} W \cos\theta \\ \frac{1}{8} WL\cos\theta \\ \frac{1}{2} W \sin\theta \\ \frac{1}{2} W \cos\theta \\ -\frac{1}{8} WL\cos\theta \end{Bmatrix} = \begin{Bmatrix} 120 \text{ kN} \\ 160 \text{ kN} \\ 2e5 \text{ kN}\cdot\text{mm} \\ 120 \text{ kN} \\ 160 \text{ kN} \\ -2e5 \text{ kN}\cdot\text{mm} \end{Bmatrix}$$

$$[k] = \begin{bmatrix} EA/L & 0 & 0 & -EA/L & 0 & 0 \\ 0 & 12EI/L^3 & 6EI/L^2 & 0 & -12EI/L^3 & 6EI/L^2 \\ 0 & 6EI/L^2 & 4EI/L & 0 & -6EI/L^2 & 2EI/L \\ -EA/L & 0 & 0 & EA/L & 0 & 0 \\ 0 & -12EI/L^3 & -6EI/L^2 & 0 & 12EI/L^3 & -6EI/L^2 \\ 0 & 6EI/L^2 & 2EI/L & 0 & -6EI/L^2 & 4EI/L \end{bmatrix}$$

$E = 20\ GPa \quad (GPa = kN/mm^2)$



$A = (150)(200) = 3e4\ mm^2$

$I_z = \dfrac{(150)(200)^3}{12} = 1e8\ mm^4$

$L^1 = L^2 = 5{,}000\ mm$

$$[k]^{1,2} = \begin{bmatrix} 120 & 0 & 0 & -120 & 0 & 0 \\ 0 & 0.192 & 480 & 0 & -0.192 & 480 \\ 0 & 480 & 1.6e6 & 0 & -480 & 0.8e6 \\ -120 & 0 & 0 & 120 & 0 & 0 \\ 0 & -0.192 & -480 & 0 & 0.192 & -480 \\ 0 & 480 & 0.8e6 & 0 & -480 & 1.6e6 \end{bmatrix}$$

transformation

$$[T] = \begin{bmatrix} c & s & 0 & 0 & 0 & 0 \\ -s & c & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$\boxed{1}$ $c = 0$, $s = 1$

$\boxed{2}$ $c = 4/5$, $s = 3/5$

$c \equiv \cos\theta = L_x/L$

$s \equiv \sin\theta = L_y/L$

(global) $\{F_f\} = [T]^T \{Q_f\}$

$$\{F_f\}^1 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \{F_f\}^2 = \begin{Bmatrix} 0 \\ 200 \\ 2e5 \\ 0 \\ 200 \\ -2e5 \end{Bmatrix} \begin{matrix} kN \\ kN \\ kN\cdot mm \\ kN \\ kN \\ kN\cdot mm \end{matrix}$$

$$[K]_{global} = [T]^T [k] [T]$$

$$[K]^1 = \begin{bmatrix} 0.192 & 0 & -480 & -0.192 & 0 & -480 \\ 0 & 120 & 0 & 0 & -120 & 0 \\ -480 & 0 & 1.6e6 & 480 & 0 & 0.8e6 \\ -0.192 & 0 & 480 & 0.192 & 0 & 480 \\ 0 & -120 & 0 & 0 & 120 & 0 \\ -480 & 0 & 0.8e6 & 480 & 0 & 1.6e6 \end{bmatrix}$$

$$[K]^2 = \begin{bmatrix} 76.87 & 57.51 & -288 & -76.87 & -57.51 & -288 \\ 57.51 & 43.32 & 384 & -57.51 & -43.32 & 384 \\ -288 & 384 & 1.6e6 & 288 & -384 & 0.8e6 \\ -76.87 & -57.51 & 288 & 76.87 & 57.51 & 288 \\ -57.51 & -43.32 & -384 & -57.51 & 43.32 & -384 \\ -288 & 384 & 0.8e6 & 288 & -384 & 1.6e6 \end{bmatrix}$$

4. Assembly
(code # method)

$$\{P_f\} = \begin{Bmatrix} F_{f_4}^1 + F_{f_1}^2 \\ F_{f_5}^1 + F_{f_2}^2 \\ F_{f_6}^1 + F_{f_3}^2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 200 \\ 2e5 \end{Bmatrix} \begin{matrix} kN \\ kN \\ kN\cdot mm \end{matrix}$$

$$[S] = \begin{bmatrix} K_{44}^1 + K_{11}^2 & K_{45}^1 + K_{12}^2 & K_{46}^1 + K_{13}^2 \\ K_{54}^1 + K_{21}^2 & K_{55}^1 + K_{22}^2 & K_{56}^1 + K_{23}^2 \\ K_{64}^1 + K_{31}^2 & K_{65}^1 + K_{32}^2 & K_{66}^1 + K_{33}^2 \end{bmatrix} = \begin{bmatrix} 77.06 & 57.51 & 192 \\ 57.51 & 163.32 & 384 \\ 192 & 384 & 3.2e6 \end{bmatrix}$$

5. Solve

$$\{P\} = \{P_f\} + [S]\{d\}$$

$$\{d\} = [S]^{-1}\{P - P_f\}$$

$$\{d\} = \begin{Bmatrix} 4.8224 \\ -2.7757 \\ -0.06246 \end{Bmatrix} \begin{matrix} mm & d_1 \\ mm & d_2 \\ rad & d_3 \end{matrix}$$

Post-Process

6. Compute member-end forces (global) : $\{F\} = \{F_f\} + [K]\{v\} \longrightarrow$ compatibility w/ $\{d\}$

$$\{v\}^1 = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \end{Bmatrix} \text{ fixed B.C.} \qquad \{v\}^2 = \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \text{ fixed B.C.}$$

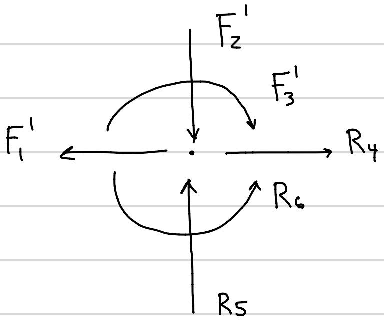
$$\{F\}^1 = \begin{Bmatrix} 29.05 \\ 333.09 \\ -47.65 \\ -29.05 \\ -333.09 \\ -97.62 \end{Bmatrix} \begin{matrix} kN \\ kN \\ kN\cdot m^* \\ kN \\ kN \\ kN\cdot m^* \end{matrix} \qquad \{F\}^2 = \begin{Bmatrix} 229.05 \\ 333.09 \\ 97.62 \\ -229.05 \\ 66.91 \\ -252.42 \end{Bmatrix} \begin{matrix} kN \\ kN \\ kN\cdot m^* \\ kN \\ kN \\ kN\cdot m^* \end{matrix}$$

* converted from kN · mm

7. Support reactions (joint equilibrium)

$$\{F\}^1 = \begin{Bmatrix} 29.05 \\ 333.09 \\ -47.65 \\ -29.05 \\ -333.09 \\ -97.62 \end{Bmatrix} \begin{matrix} \text{kN} \\ \text{kN} \\ \text{kN}\cdot\text{m}^* \\ \text{kN} \\ \text{kN} \\ \text{kN}\cdot\text{m}^* \end{matrix} \qquad \{F\}^2 = \begin{Bmatrix} 229.05 \\ 333.09 \\ 97.62 \\ -229.05 \\ 66.91 \\ -252.42 \end{Bmatrix} \begin{matrix} \text{kN} \\ \text{kN} \\ \text{kN}\cdot\text{m}^* \\ \text{kN} \\ \text{kN} \\ \text{kN}\cdot\text{m}^* \end{matrix}$$

1

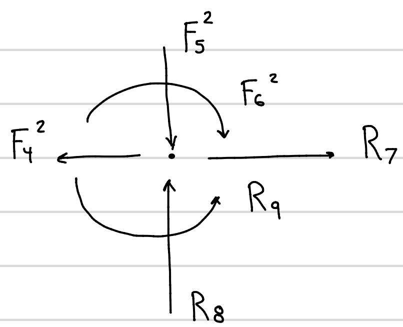


$\sum F_x = 0 \quad R_4 = F_1^1 = 29.05 \text{ kN}$

$\sum F_y = 0 \quad R_5 = F_2^1 = 333.09 \text{ kN}$

$\sum M_\cdot = 0 \quad R_6 = F_3^1 = -47.65 \text{ kN}\cdot\text{m} \quad \therefore \curvearrowright$

3



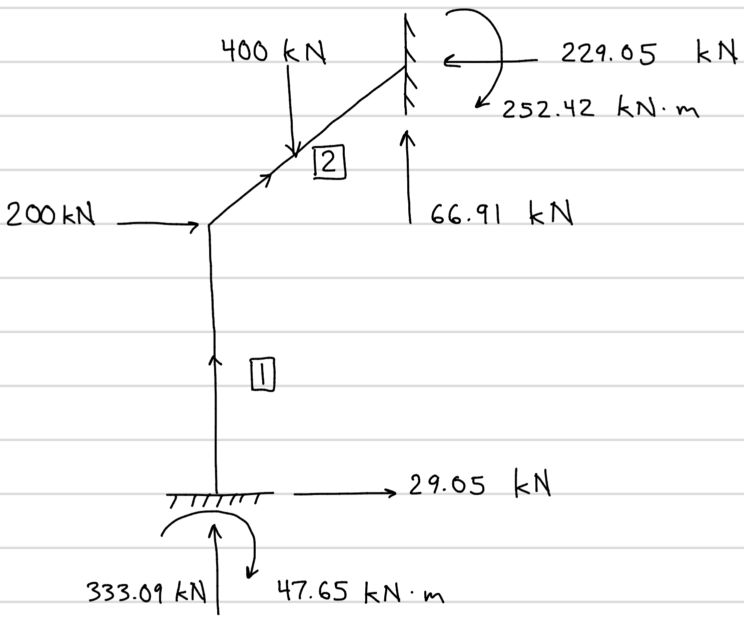
$\sum F_x = 0 \quad R_7 = F_4^2 = -229.05 \text{ kN} \quad \therefore \leftarrow$

$\sum F_y = 0 \quad R_8 = F_5^2 = 66.91 \text{ kN}$

$\sum M_\cdot = 0 \quad R_9 = F_6^2 = -252.42 \text{ kN}\cdot\text{m} \quad \therefore \curvearrowright$

* Check overall equilibrium *

8. Draw axial force, shear force, and bending moment diagrams



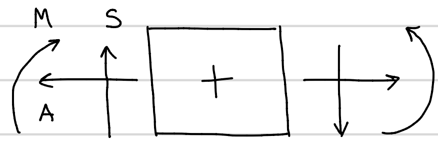
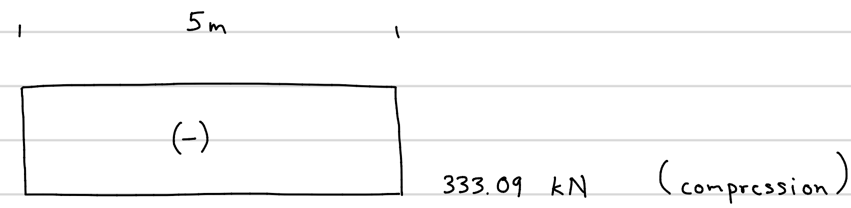


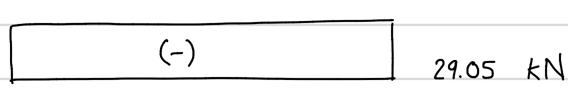
diagram positive sign convention

Member 1

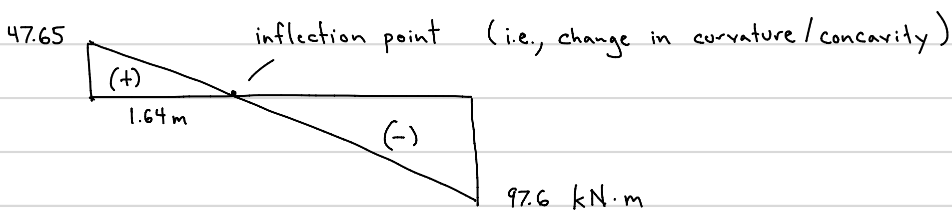
Axial



Shear

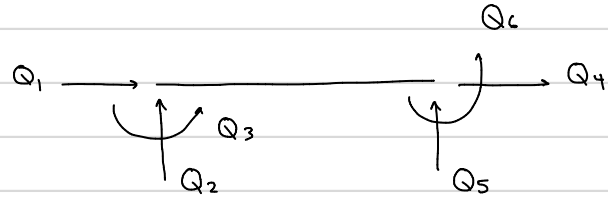


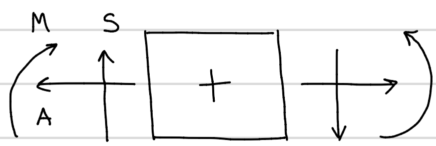
Moment



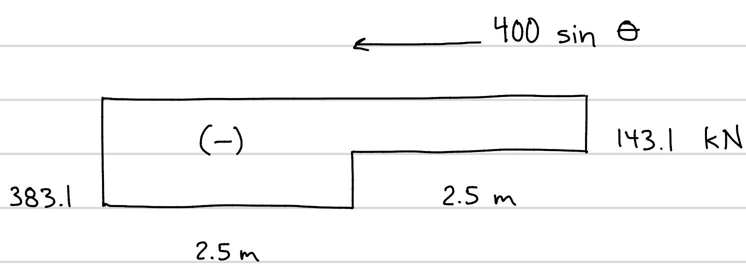
Member $\boxed{2}$ (easier to work with Q) $\{Q\} = [T]\{F\}$

$$\{Q\}^2 = \begin{Bmatrix} 383.1 \\ 129.04 \\ 97.6 \\ -143.1 \\ 191 \\ -252.42 \end{Bmatrix} \begin{array}{ll} \text{kN} & \text{axial} \\ \text{kN} & \text{shear} \\ \text{kN}\cdot\text{m} & \text{moment} \\ \text{kN} & \\ \text{kN} & \\ \text{kN}\cdot\text{m} & \end{array}$$



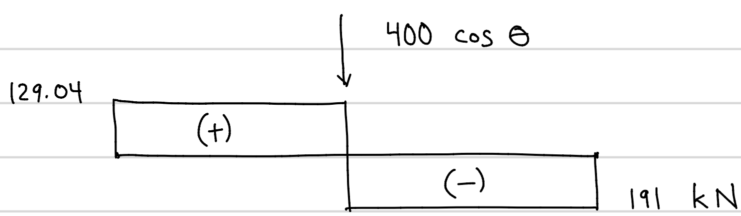


Axial

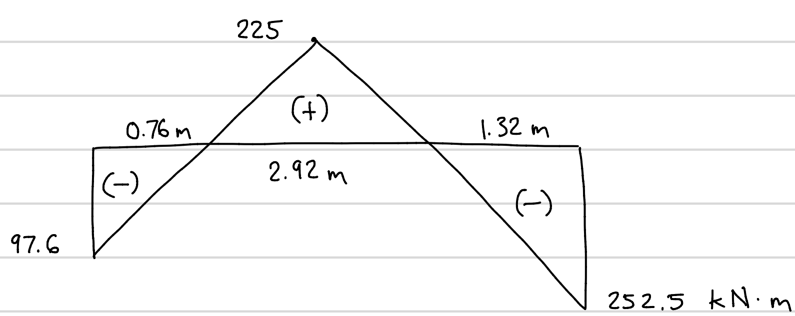


* don't forget interior loads *

Shear



Moment

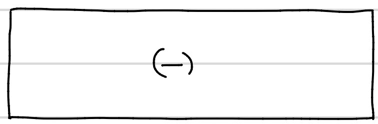


9. Calculate stresses

Member $\boxed{1}$

Axial Stress

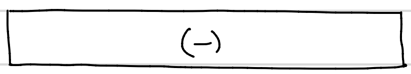
$\sigma_a = {}^{Force}/_{Area}$



11.1 MPa

Area = $3e4\ mm^2$

Shear Stress

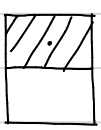


1.45 MPa

$\tau = \frac{VQ}{Ib}$

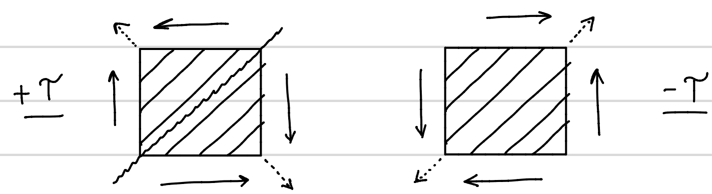
* sign of shear stress important when:
  - combining w/ torsion effects
  - anisotropic materials (e.g. fiber-composites)

$I = 1e8\ mm^4$



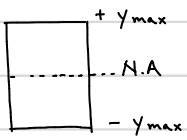
$Q = (150 \cdot 100)(50)$

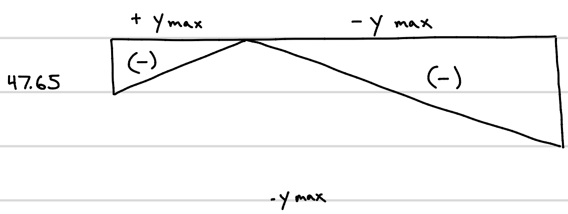
$= 7.5e5\ mm^3$



Bending Stress

$\sigma_b = \frac{-My}{I}$ * $\sigma = \sigma_a + \sigma_b$ $\therefore$ we need to consider worst case combination(s)





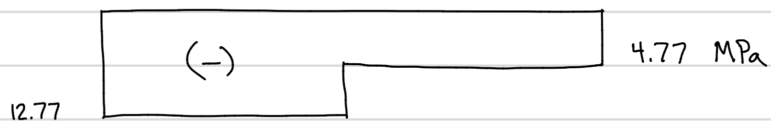
* $\sigma_{max} = -11.1 - 97.65 = -108.75$ MPa (compression) * $\tau_{max} = 1.45$ MPa

σ_{max} (tension) $= -11.1 + \underset{+y_{max}}{97.65} = 86.55$ MPa

Member $\boxed{2}$

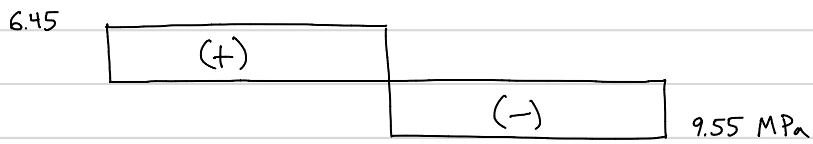
Axial Stress

$$\sigma_a = \frac{P}{A}$$



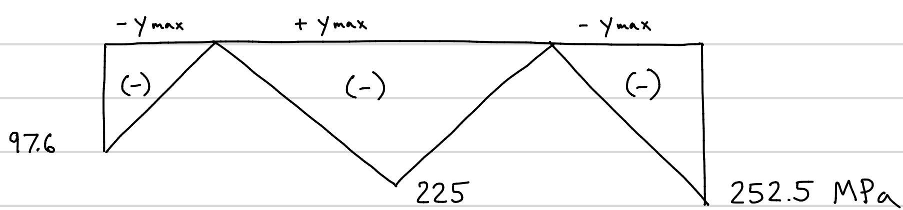
Shear Stress

$$\tau = \frac{VQ}{Ib}$$



Bending Stress

$$\sigma_b = -\frac{My}{I}$$



$\sigma_{max} =$ 257.27 MPa (compression) $\qquad \tau_{max} =$ 9.55 MPa