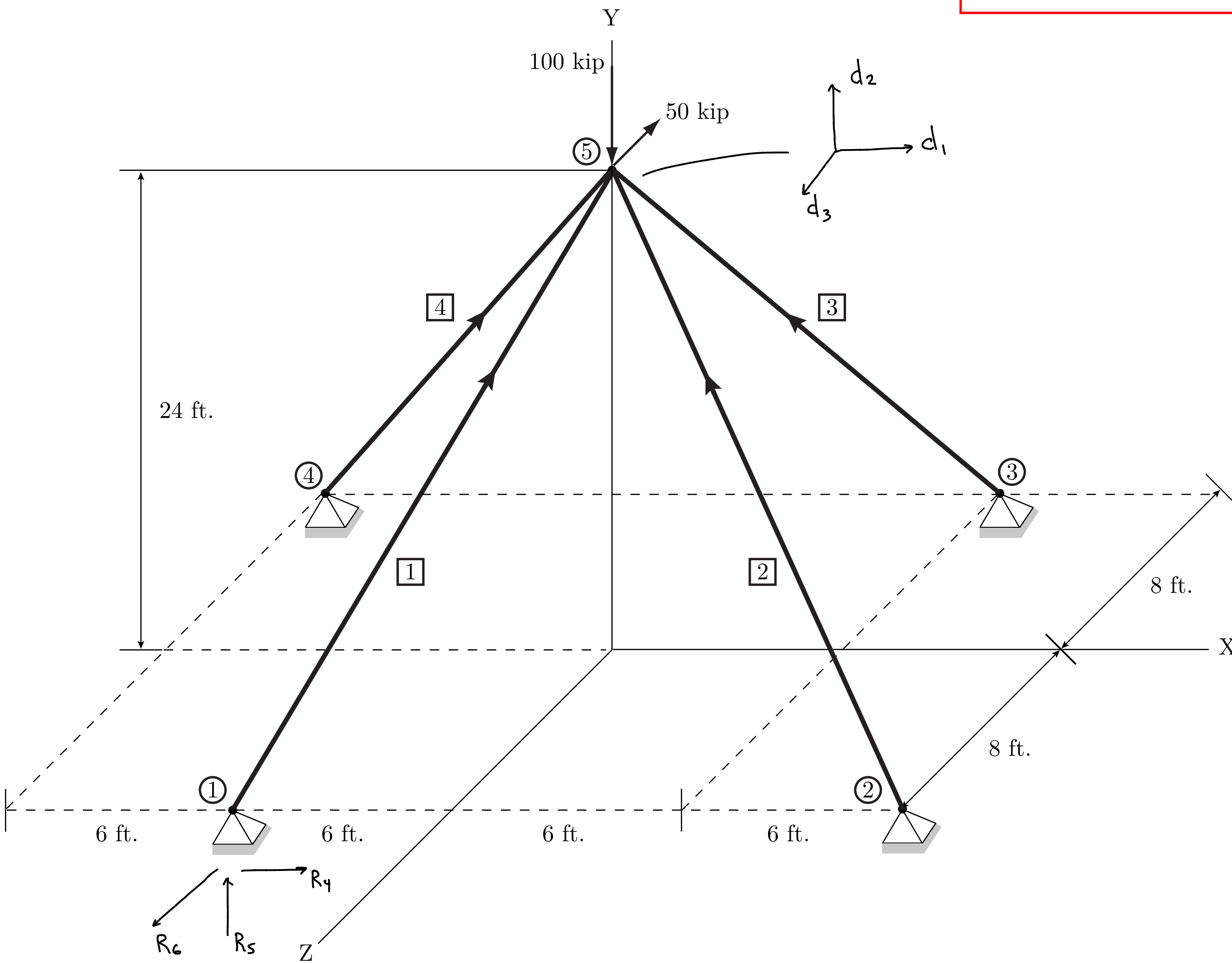
Space Truss Example



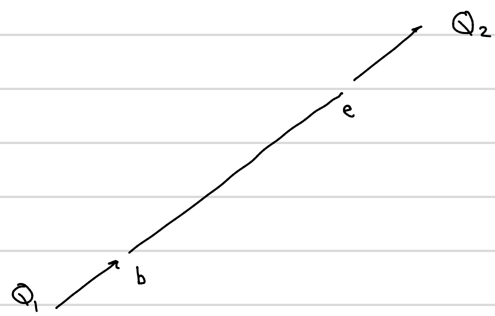
E = 10e3 ksi ; A = 8.4 in^2 for all members

1. Label structure: joints, members, axes (local/global), DOFs, reactions

2. Create structural force vector $\{P\}_{3\times1}$

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -100 \\ -50 \end{Bmatrix} \text{ kip}$$

3. Determine local [k] and transformation matrix [T]



$$[k] = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$[T] = \begin{bmatrix} \cos\theta_x & \cos\theta_y & \cos\theta_z & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta_x & \cos\theta_y & \cos\theta_z \end{bmatrix}$$

$$\cos\theta_x = \frac{L_x}{L} \qquad \cos\theta_y = \frac{L_y}{L} \qquad \cos\theta_z = \frac{L_z}{L}$$

$$L = \sqrt{\underbrace{(X_e - X_b)^2}_{L_x} + \underbrace{(Y_e - Y_b)^2}_{L_y} + \underbrace{(Z_e - Z_b)^2}_{L_z}}$$

	L	$\frac{EA}{L}$	$\cos\theta_x$	$\cos\theta_y$	$\cos\theta_z$
1	312 in.	269.23 k/in	6/26	24/26	-8/26
2	336 in.	250.00	-12/28	24/28	-8/28
3	312 in.	269.23	-6/26	24/26	8/26
4	336 in.	250.00	12/28	24/28	8/28

Compute global $[K] = [T]^T [k] [T]$

4. Assemble $[S]$ using code # method

code # / member #	F_1, V_1	F_2, V_2	F_3, V_3	F_4, V_4	F_5, V_5	F_6, V_6
1	4	5	6	1	2	3
2	7	8	9	1	2	3
3	10	11	12	1	2	3
4	13	14	15	1	2	3

$$[K] = \begin{array}{c} \\ a \\ b \\ c \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccccc} a & b & c & 1 & 2 & 3 \end{array} \\ \left[\begin{array}{ccc|ccc} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ \hline K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{array}\right] \end{array}$$

$$[S] = \sum_{i=1}^{4} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} K_{44}^i & K_{45}^i & K_{46}^i \\ K_{54}^i & K_{55}^i & K_{56}^i \\ K_{64}^i & K_{65}^i & K_{66}^i \end{bmatrix} \end{array} \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}$$

5. Solve $\{P\} = [S]\{d\}$

$$\{d\} = [S]^{-1}\{P\}$$

$$\{d\} = \begin{Bmatrix} 0.10913 \\ -0.12104 \\ -0.57202 \end{Bmatrix} \text{ in.}$$

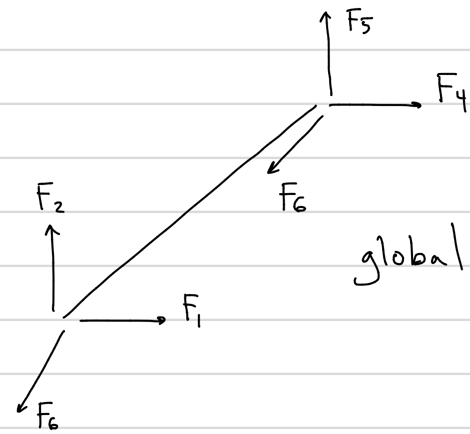
Post-Processing

6. Compute global member end forces

$$\{F\} = [K]\{v\}$$ use compatibility to determine $\{v\}$ from $\{d\}$

$$\{v\}^{1-4} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ d_1 \\ d_2 \\ d_3 \end{Bmatrix}$$

(first three entries: pin support)

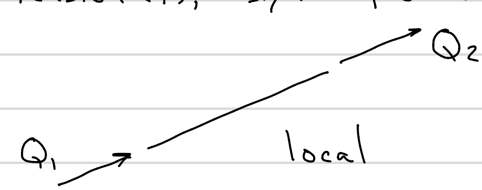


$$\{F\}^1 = \begin{Bmatrix} -5.56 \\ -22.23 \\ 7.41 \\ 5.56 \\ 22.23 \\ -7.41 \end{Bmatrix} \text{kip} \quad \{F\}^2 = \begin{Bmatrix} 1.38 \\ -2.77 \\ 0.92 \\ -1.38 \\ 2.77 \\ -0.92 \end{Bmatrix} \quad \{F\}^3 = \begin{Bmatrix} -19.44 \\ 77.77 \\ 25.92 \\ 19.44 \\ -77.77 \\ -25.92 \end{Bmatrix} \quad \{F\}^4 = \begin{Bmatrix} 23.62 \\ 47.23 \\ 15.74 \\ -23.62 \\ -47.23 \\ -15.74 \end{Bmatrix}$$

7. Calculate axial bar forces $\{Q\} = [T]\{F\}$ 3D/2D truss: report Q_2/Q_3

+ sign tension (T), − sign compression (C)

*Can also use Pythagorean theorem,
e.g. $Q_1 = \sqrt{F_1^2 + F_2^2 + F_3^2}$, however,
direction(s) harder to determine for tension/compression



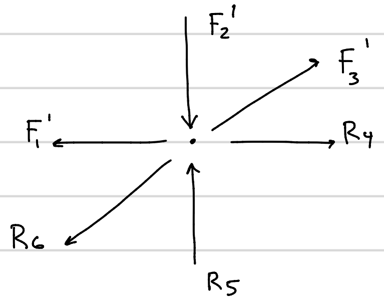
1. $Q_2^1 = 24.085$ kip (T) $\sigma_a^1 = 2.867$ ksi
2. $Q_2^2 = 3.2289$ k (T) $\sigma_a^2 = 0.384$ ksi
3. $Q_2^3 = -84.248$ k (C) $\sigma_a^3 = -10.03$ ksi
4. $Q_2^4 = -55.104$ k (C) $\sigma_a^4 = -6.56$ ksi

$\sigma_{axial} = \text{Force}/\text{Area}$ $A = 8.4$ in.^{2}

* compare applied stress to material strength

8. Determine support reactions from joint equilibrium

joint ①



$\Sigma F_x = 0$ $\quad R_4 = F_1' = -5.56$ k $\quad \therefore \quad 5.56$ k $\leftarrow$ (-x)

$\Sigma F_y = 0$ $\quad R_5 = F_2' = -22.23$ k $\quad \therefore \quad 22.23$ k $\downarrow$ (-Y)

$\Sigma F_z = 0$ $\quad R_6 = F_3' = 7.41$ k $\quad \therefore \quad 7.41$ k $\swarrow$ (+Z)

Similarly for other reactions at supports/joints

②	③	④
$R_7 = 1.38$	$R_{10} = -19.44$	$R_{13} = 23.62$
$R_8 = -2.77$	$R_{11} = 77.77$	$R_{14} = 47.23$
$R_9 = 0.92$	$R_{12} = 25.92$	$R_{15} = 15.74$

* Can check overall equilibrium of structure to verify correct solution has been obtained