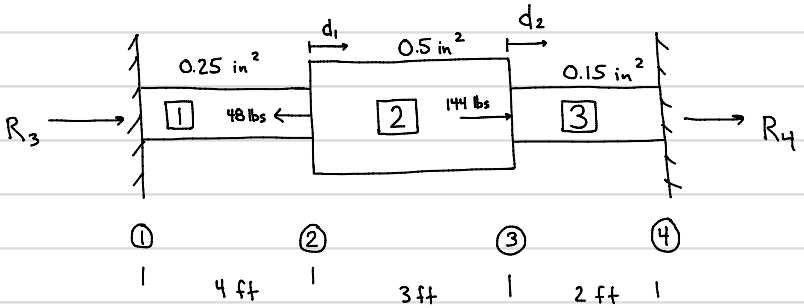
Fall 2024
Matrix Displacement Method
Uniaxial Structures
Part III - Numerical Example



1. Find Displacements $\{p\} = [s]\{d\}$
2. Member Forces
3. Reactions

$E = 30e3$ ksi

1. $\{p\} = \begin{Bmatrix} P_1 \\ P_2 \end{Bmatrix} = \begin{Bmatrix} -48 \\ 144 \end{Bmatrix}$ lbs

$[S]$ → code # assembly to get $[S]$ from member k_{ij}s

member # \ code #	M_1, Q_1	M_2, Q_2
1	3	1
2	1	2
3	2	4

M_1, Q_1 → M_2, Q_2

$$[S] = \begin{matrix} \\ 1 \\ 2 \end{matrix} \begin{matrix} \begin{matrix} d_1 \\ 1 \end{matrix} & \begin{matrix} d_2 \\ 2 \end{matrix} \\ \begin{bmatrix} k'_{22} + k_{11}^2 & k_{12}^2 \\ k_{21}^2 & k_{22}^2 + k_{11}^3 \end{bmatrix} \end{matrix} = \begin{bmatrix} 572917 & -416667 \\ -416667 & 604167 \end{bmatrix}$$

1. $[k]^1 = \begin{matrix} \\ 3 \\ 1 \end{matrix} \begin{matrix} \begin{matrix} 3 & 1 \end{matrix} \\ \begin{bmatrix} k'_{11} & k'_{12} \\ k'_{21} & k'_{22} \end{bmatrix} \end{matrix}$

2. $[k]^2 = \begin{matrix} \\ 1 \\ 2 \end{matrix} \begin{matrix} \begin{matrix} 1 & 2 \end{matrix} \\ \begin{bmatrix} k_{11}^2 & k_{12}^2 \\ k_{21}^2 & k_{22}^2 \end{bmatrix} \end{matrix}$

3. $[k]^3 = \begin{matrix} \\ 2 \\ 4 \end{matrix} \begin{matrix} \begin{matrix} 2 & 4 \end{matrix} \\ \begin{bmatrix} k_{11}^3 & k_{12}^3 \\ k_{21}^3 & k_{22}^3 \end{bmatrix} \end{matrix}$

$$[k] = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$$

	E	A	L	EA/L
1	30e6	0.25	48	156250
2	30e6	0.5	36	416667
3	30e6	0.15	24	187500
	psi	in^2	in.	lbs/in

* consistent units *

$$\{P\} = [S]\{d\} \qquad \{d\} = [S]^{-1}\{P\}$$

$$\begin{Bmatrix} -48 \\ 144 \end{Bmatrix} = \begin{bmatrix} 572917 & -416667 \\ -416667 & 604167 \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} \qquad \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} = \begin{Bmatrix} 1.7968 \\ 3.6226 \end{Bmatrix} \times 10^{-4} \text{ in.}$$

Solution step

↓

Post-processing

2. Member Forces

$$\{Q\} = [k]\{u\} \qquad \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

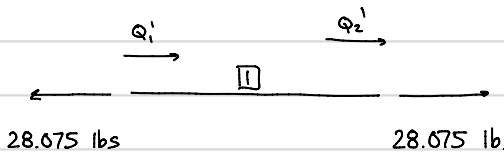
[1] $$\begin{Bmatrix} Q_1' \\ Q_2' \end{Bmatrix} = 156\,250 \text{ lbs/in} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} u_1' \\ u_2' \end{Bmatrix}$$

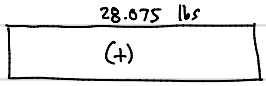
$u_1' \rightarrow$ 0 support B.C.

$u_2' \rightarrow d_1 = 1.7968 \times 10^{-4}$ in.

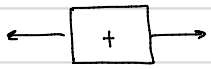
compatibility !!

$$\begin{Bmatrix} Q_1' \\ Q_2' \end{Bmatrix} = \begin{Bmatrix} -28.075 \\ 28.075 \end{Bmatrix} \text{ lbs}$$





axial force diagram



axial force

Same process for [2], [3]

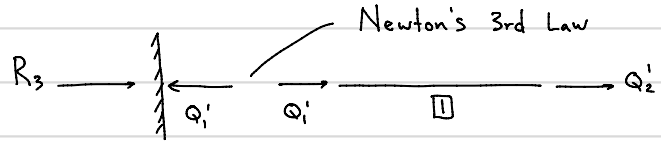
$$\begin{Bmatrix} Q_1^2 \\ Q_2^2 \end{Bmatrix} = \begin{Bmatrix} -76.075 \\ 76.675 \end{Bmatrix} \text{ lbs} \quad \text{tension}$$

$$\begin{Bmatrix} Q_1^3 \\ Q_2^3 \end{Bmatrix} = \begin{Bmatrix} 67.925 \\ -67.925 \end{Bmatrix} \text{ lbs} \quad \text{compression}$$

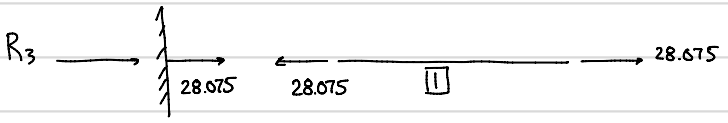
sign of Q_2 indicates tension/compression

(+) (−)

3. Find Reactions (joint equilibrium)



$\Sigma F_x = 0 \quad R_3 - Q_1' = 0 \quad \therefore \quad R_3 = Q_1' = -28.075$

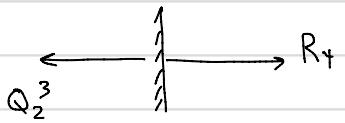


$\Sigma F_x = 0 \quad R_3 + 28.075 = 0 \quad \therefore \quad R_3 = -28.075$

$R_3 = 28.075$ lbs ←

Determining R_4:

Option 1 - joint equilibrium



$\Sigma F_x = 0 \quad R_4 - Q_2^3 = 0 \quad \therefore \quad R_4 = Q_2^3 = -67.925$

$R_4 = 67.925$ lbs ←

Option 2 - overall equilibrium

$\Sigma F_x = 0 \quad R_3 + R_4 + P_1 + P_2 = 0$

$-28.075 + R_4 - 48 + 144 = 0$

$R_4 = -67.925$

* overall equilibrium good final check